The Use of a Simplified Markowitz Model in Choosing a Profitable Portfolio

Alexandru Cristian Dobre
University of Petrosani, Romania
my_xpress_2005@yahoo.com

Abstract

The aim of this research is to improve investment processes in the capital market, so that the decisions taken are the best, at the best time and at the best price in line with the risk factor. In this paper we present a simplified model of Harry Markowitz, using Microsoft Excel. By gathering public data on the shares of selected companies as being in the current economic trend, over a period of 6 months, we are able to make a prediction of the amount of these shares in a profitable portfolio using the Markowitz model.

For the portfolio manager, it is sufficient for the client investor of the respective investment fund to formulate concretely its requirements regarding the minimum accepted return level and the model allows the identification of such a portfolio structure that determines the obtaining of this level of the indicator in minimum risk conditions.

Key words: portfolio, Markowitz, return
J.E.L. classification: G11

1. Introduction

One of the most scientifically recognized models of finance is the one developed by Harry Markowitz [1952] considered by many specialists as the founder of modern finance. The model offers a solution for the efficient management of the portfolio, for the rational investors characterized by risk aversion (quantified by dispersion of the expected return), in the conditions in which they request to obtain a certain return.

In order to identify the levels associated with the estimated profitability and the risk associated with this anticipation, one can use either the scenario technique or one can accept the strong static hypothesis.

It must be remembered that the modern theory of the portfolio owes to the American professor the definition of risk through the dispersion of profitability. Specifically, the portfolio manager associates with the future evolution of a security a certain return and a certain risk, quantified by the quadratic mean deviation of this return. Based on these indicators, a certain structure of the portfolio can be determined so as to obtain a level of return (the one requested by the client investor) in the conditions of a minimum risk or, analogously, to obtain a maximum return in the conditions of an accepted risk.

2. Theoretical background

The field of the capital market has a vast specialized literature. We will remember the most important works that helped us to elaborate this work in the rows below.

Livia Ilie published in 2007 at the Continent Bucharest Publishing House, the book entitled "Piața de capital".

In 2008, Duțescu Cristian published at C.H. Bech, Bucharest, the book with the title "Manipularea pieței de capital".

In 2011, Dobre Baron Oana, Muntean Liliana published at the Petroșani University Publishing House, the book entitled " Gestiunea portofoliilor de titluri ".

Lipara Carmen published in 2012 at ASE Bucharest Publishing House, the book entitled "Evaluarea acțiunilor: o abordare din perspectiva teoriilor comportamentale ".

Tutuianu Adrian published in 2007 at Hamangiu Publishing House Bucharest, the book entitled "Piața de capital. Regimul juridic aplicabil participanților ".

In the following we will also mention important foreign authors.

Markowitz M. Harry published the book "Portfolio selection: efficient diversification of investments" at Blackwell Publishing House, Malden, in 1991.

Daniel A. Strachman published in 2002 at Wiley Publishing House, the book entitled "Essential Stock Picking Strategies".

Brentani Christine published in 2004 at Elsevier Publishing House, Oxford, the book entitled "Portfolio management in practice".

To describe the Markowitz model hypothesis exemplified in this paper we can list three important criteria of the Markowitz model:

a) The selection criterion for effective combinations of "n" securities is "hope - dispersion" (return-risk).

b) All the "n" securities are risky, characterized by a certain hope of profitability (Ei), dispersion (σii) and covariation with each of the other securities in the portfolio (σij).

c) Expected return on portfolio (E*p) it is a variable exogenous to the model, being provided by investors.

The structure of the portfolios on the Markowitz (Markowitz, 1991, p. 281) border will be determined by minimizing the risk of the portfolio under some restrictions determined by the model assumptions:

- Budgetary constraint, which requires that all invested capital be included in securities purchased from the portfolio: $\sum x_i = 1$ (1)
- The performance requirement of the portfolio, respectively, a certain expected profitability E*p, which is the average expected return on the securities in the portfolio: $\sum x_i E_i = E_p$ (2)
- Portfolio risk minimization can be done through the combined Lagrange function:

$$L = \frac{1}{2}\sum_i \sum_j x_i x_j \sigma_{ij} + \lambda_1\left(\sum_i x_i E_i - E^*_p\right) + \lambda_2\left(\sum_i x_i - 1\right) \quad (3)$$

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} & E_1 & 1 \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} & E_2 & 1 \\ \vdots & & & & & \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 & E_n & 1 \\ E_1 & E_2 & \cdots & E_n & 0 & 0 \\ 1 & 1 & \ldots & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ E^*_p \\ 1 \end{bmatrix} \Leftrightarrow X = W^{-1} \times K \quad (4)$$

W x X = K

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} & 1 \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} & 1 \\ \vdots & & & & \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 & 1 \\ 1 & 1 & \ldots & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \Leftrightarrow X = W^{-1} \times K \quad (5)$$

W x X = K

The data is used to determine the composition of PVMA.

Thus, the weights of the various securities in the expected return portfolio become known E*p located on the efficient Markowitz border and the risk of this portfolio can also be found.

$$\sigma^2{}_p = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \quad (6)$$

3. Research methodology

The methodology usedin this paper is based on documentary, bibliographic and theoretical research, as well as analyzes from practical, specific activity in the field of capital markets, at national and European level. During the research, qualitative research methods were used to collect data, such as observation and case study. At the same time, quantitative methods based on deductive processes were used, through which theories were verified, producing results that could be generalized. Thus, starting from the identification of the problems and to their evaluation, it is necessary to have a complementarity between the quantitative and qualitative methods, because with the help of the case study and the studied content analyzes the obtained results can be developed, disseminated and consulted.

To simulate a portfolio in our case study we chose the companies BRD Groupe Societe Generale S.A., Banca Transilvania S.A., Turbomecanica S.A. and S.N. Nuclearelectrica S.A. The companies were chosen from different sectors for a wider diversification of the model.

In the first stage, the quotations of the companies were collected for a period of 6 months.

In the next step, the daily quotation yields were calculated in order to calculate an average return on the shares of each company.

Following the construction of the covariance matrix, the inverse of the covariance matrix was then performed in order to find out the minimum variance point of the portfolio chosen by us.

A simulation of the proposed returns was performed in accordance with the point of minimum variance in order to achieve a profitable portfolio.

Following the model, some simulations were performed with a proposed return of 35%, 55% and 75%, finding the share of each element in the portfolio that is necessary for the investment in question.

The formulas used in Microsoft Excel for the values obtained were:

E=(E1-E2)/E3

=covar()

=minverse()

=mmult()

=average()

Table no. 1. Markowitz Model

	Rentabilittate medie E						
BRD	TLV	TBM	SNN				
-0,11%	-0,05%	0,23%	0,24%				
	Matrice de covarianță						
	BRD	TLV	TBM	SNN			
BRD	0,000816	0,000722138	0,000163001	0,000236604	1		
TLV	0,000722	0,000923436	0,000210231	0,000271722	1		
TBM	0,000163	0,000210231	0,000392937	0,000155089	1		
SNN	0,000237	0,000271722	0,000155089	0,00055912	1		
	1	1	1	1	0		
	Invers matrice de covarianță						
BRD	3832,098	-3022,849526	-359,5456119	-449,7032697	0,221885752		0
TLV	-3022,85	3706,695573	-421,9790343	-261,8670133	-0,08011339		0
TBM	-359,5456	-421,9790343	1968,204912	-1186,680266	0,557980656		0
SNN	-449,7033	-261,8670133	-1186,680266	1898,250549	0,300246982		0
	0,221886	-0,08011339	0,557980656	0,300246982	-0,000285141		1
	Punctul de variație minimă absolută				PVMA		
BRD	0,221886	0,221885752	0,221885752	0,221885752	0,221885752	x1	
TLV	-0,080113	-0,08011339	-0,08011339	-0,08011339	-0,08011339	x2	
TBM	0,557981	0,557980656	0,557980656	0,557980656	0,557980656	x3	
SNN	0,300247	0,300246982	0,300246982	0,300246982	0,300246982	x4	
	-0,000285	-0,000285141	-0,000285141	-0,000285141	-0,000285141		
	Simulare rentabilități propuse						
	Ep	x1	x2	x3	x4		
Epvma=	0,18%	22,18%	-8,01%	55,79%	30,02%		
Em=	0,54	34,17%	3,69%	85,93%	46,24%		
	1,00%	22,41%	7,93%	56,36%	30,32%		
	1,05%	22,42%	7,93%	30,34%	30,34%		
	3%	22,85%	7,77%	57,47%	30,93%		
	Modelul markowitz in formă matriceală						
	0,000816	0,000722138	0,000163001	0,000236604	0,54	1	
	0,000722	0,000923436	0,000210231	0,000271722	1	1	
	0,000163	0,000210231	0,000392937	0,000155089	1,05	1	
	0,000237	0,000271722	0,000155089	0,00055912	3	1	
	0,54	1	1,05	3	0	0	
	1	1	1	1	0	0	

Source: realized by the author based on data from www.bvb.ro.

Table no. 2.Continuation of the Markowitz Model

	Rentabilitate propusă 0.35%							
	3035,333	-2771,443091	-986,6837271	722,7937084	-0,297283946	0,675637835		0
	-2771,443	3627,368329	-224,0957202	-631,8295179	0,093803153	-0,223287539		0
	-986,6837	-224,0957202	1474,581235	-263,8017881	-0,233993742	0,915131284		0
	722,7937	-631,8295179	-263,8017881	172,8375976	0,437474536	-0,36748158		0
	-0,297284	0,093803153	-0,233993742	0,437474536	-0,000110921	0,000169301		0,35
	0,675638	-0,223287539	0,915131284	-0,36748158	0,000169301	-0,00054355		1
	Rentabilitate propusă 0.35%							
BRD	0,571588	0,571588454	0,571588454	0,571588454	0,571588454	0,571588454	x1	
TLV	-0,190456	-0,190456435	-0,190456435	-0,190456435	-0,190456435	-0,190456435	x2	
TBM	0,833233	0,833233474	0,833233474	0,833233474	0,833233474	0,833233474	x3	
SNN	-0,214365	-0,214365493	-0,214365493	-0,214365493	-0,214365493	-0,214365493	x4	
	0,00013	0,000130479	0,000130479	0,000130479	0,000130479	0,000130479		
	-0,000484	-0,000484294	-0,000484294	-0,000484294	-0,000484294	-0,000484294		
	Rentabilitate propusă 0.55%							
	3035,333	-2771,443091	-986,6837271	722,7937084	-0,297283946	0,675637835		0
	-2771,443	3627,368329	-224,0957202	-631,8295179	0,093803153	-0,223287539		0
	-986,6837	-224,0957202	1474,581235	-263,8017881	-0,233993742	0,915131284		0
	722,7937	-631,8295179	-263,8017881	172,8375976	0,437474536	-0,36748158		0
	-0,297284	0,093803153	-0,233993742	0,437474536	-0,000110921	0,000169301		0,55
	0,675638	-0,223287539	0,915131284	-0,36748158	0,000169301	-0,00054355		1
	Rentabilitate propusă 0.55%							
BRD	0,512132	0,512131664	0,512131664	0,512131664	0,512131664	0,512131664	x1	
TLV	-0,171696	-0,171695804	-0,171695804	-0,171695804	-0,171695804	-0,171695804	x2	
TBM	0,786435	0,786434725	0,786434725	0,786434725	0,786434725	0,786434725	x3	
SNN	-0,126871	-0,126870585	-0,126870585	-0,126870585	-0,126870585	-0,126870585	x4	
	0,000108	0,000108295	0,000108295	0,000108295	0,000108295	0,000108295		
	-0,00045	-0,000450434	-0,000450434	-0,000450434	-0,000450434	-0,000450434		
	Rentabilitate propusă 0.75%							
	3035,333	-2771,443091	-986,6837271	722,7937084	-0,297283946	0,675637835		0
	-2771,443	3627,368329	-224,0957202	-631,8295179	0,093803153	-0,223287539		0
	-986,6837	-224,0957202	1474,581235	-263,8017881	-0,233993742	0,915131284		0
	722,7937	-631,8295179	-263,8017881	172,8375976	0,437474536	-0,36748158		0
	-0,297284	0,093803153	-0,233993742	0,437474536	-0,000110921	0,000169301		0,75
	0,675638	-0,223287539	0,915131284	-0,36748158	0,000169301	-0,00054355		1
	Rentabilitate propusă 0.75%							
BRD	0,452675	0,452674875	0,452674875	0,452674875	0,452674875	0,452674875	x1	
TLV	-0,152935	-0,152935174	-0,152935174	-0,152935174	-0,152935174	-0,152935174	x2	
TBM	0,739636	0,739635977	0,739635977	0,739635977	0,739635977	0,739635977	x3	
SNN	-0,039376	-0,039375678	-0,039375678	-0,039375678	-0,039375678	-0,039375678	x4	
	8,61E-05	8,61106E-05	8,61106E-05	8,61106E-05	8,61106E-05	8,61106E-05		
	-0,000417	-0,000416574	-0,000416574	-0,000416574	-0,000416574	-0,000416574		

Source: realized by the author based on data from www.bvb.ro.

In a financial market where the criterion of risk-return is the one accepted in the choice of investments, Markowitz points out that all investors, as long as they are rational and characterized by risk aversion, will invest in a set of portfolios characterized by a specific risk-return relationship. (Raj S., 2019, p. 233)

As the American professor, winner of the Nobel Prize for Economics in 1990, points out, the portfolio selection process can be structured in two stages. The first stage begins with the observations made on the securities and ends with the estimation of future performances associated with the securities available on the financial market. The second stage starts from these values associated with future performance indicators and ends with the selection of a specific portfolio.

4. Findings

Therefore, following the elaboration of the model, the values of the weights in the portfolio of the shares x1, x2, x3, x4, are presented in the last summary table no. 3

Table no. 3. Weights to simulate expected profitability

Rent aștep. E*	x1	x2	x3	x4
35,00%	57,16%	-19,05%	83,32%	-21,44%
55,00%	51,21%	-17,17%	78,64%	-12,69%
75,00%	45,27%	-15,29%	73,96%	-3,94%

Source: realized by the author based on data from www.bvb.ro.

In order to identify the levels associated with the estimated profitability and the risk associated with this anticipation, one can use either the scenario technique or one can accept the strong static hypothesis. The first method has the advantage of a possible more adequate perception of the reality on the financial market on which the securities in which it invests operate, but creates difficulties for the manager in arguing his decisions, characterized in this case by a significant degree of subjectivism. Given the decision-making under the strong static hypothesis, although decisions can be considered "objective", in the sense that they can be easily justified, we can not but raise the issue of the relevance of past indicators on predictions, obviously related to the future. (Waldemar et al, 2019, p. 125)

A solution could be given by estimating a certain future return and quantifying the risk associated with this estimate by dispersing or deviating the square mean of the historical returns over an acceptable time frame in terms of modeling needs.(Robert G., 2015, p. 178)

5. Conclusions

Developing a solid investment process is a personal challenge for investors. To create an effective investment plan, the investor needs to look to the best investors in the world, the true masters. They must be chosen and studied with great care; reading and listening to them and trying to gather as much as possible from them. This is how a successful investor approaches this vast economic field. This approach is refined by the experience of the investor's daily life that helps him in his investment approach. This is how the investor develops strong and coherent beliefs that allow investors to be brave and turn their backs on the herd, being patient and accepting uncertainty and finally keeping their minds clear when those around them are agitated.

Markets work reasonably well because they are just exchanges between people who act freely. Some market participants or others will always recognize when there is a disorder in the market, and through their actions they will address it.

It provides us with a basis for approaching the investment process, which aims to add value that others will not see or cannot see. Real estate assets, especially the shares of listed companies, are the best for maintaining the purchasing power of our savings in the long run.

6. References

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