Optimizing a Portfolio Depending on the Attitude Towards Risk

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Abstract

The risk is associated with the profitability of any security as well as with any economic activity. The actors of the financial market are differently placed in front of the risks, according to a series of criteria such as trading experience, the available capital, real time information possibilities, the ability to interprret the data, the intuition, etc. The psychological factor always had an essential role in taking certain risk doses. The analysis of the period prior to the recent financial crisis showed that an important number of the investors deliberately omitted the risk exposure considering wrongly that the level of the present financial market eliminates the possibility of a crisis to appear. The reality proved the opposite, reason for which we considered it necessary to present a few mathematical models to appreciate the attitude in front of risk. With this paper, the authors aim to point out that reality has proven the opposite, which is why we considered it necessary to present some mathematical models for assessing the attitude towards risk.

Key words: coefficient β, portfolio of risky actions, risk-averse portfolio, variation of the return and the risk in relation to the weight of securities
J.F.L. classification: F30, G01

1. Introduction

Selecting the securities to be included in the portfolio was based on coefficient β, on the previous return and individual attitude towards risk. Both portfolios contain ATB security bond and there have been taken into account the same possibilities for the manifestation of the possible stages, so in the end to be able to compare the returns and risks of the different attitudes towards risk. The expected return of the security bonds for each possible stage of the economic environment has been established pursuant to the return and the standard deviation for the three possible stages of the market: stagnation ($\overline{R}$); growth ($\overline{R}$ + σ); recession ($\overline{R}$ - σ) [1]. There has been ruled out the possibility of new information for regarding the activity of the issuers, analysis being done by extrapolating historical data. The manifestation probabilities of the possible event stages are subjectively determined, based on personal assessment of the impact on the stock market given the fact that Romanian has entered the European Union. Using other pairs of probabilities will change the levels of return and portfolio risk and also its structure. In order to demonstrate it we have taken into account two extreme portfolios depending on their attitude towards risk, with specific selection criteria, among which you can make an infinite number of optimum combination with the traded securities.

2. Theoretical background - Portfolio of risky actions

The highest average return was the selection criterion, and we have taken into consideration ATB and OLT securities. Both security bonds have a supra-unitary volatility of 1.25, namely 1.14, which allows a multiplication of return in terms of market growth, but also an increased risk if the

trend is reversed. The correlation with the market of the two securities is positive and of high intensity (0.75 for ATB and 0.66 for OLT), which indicates a dependence on the market conditions.

Table no. 1. The calculation of the dispersion and covariance

The possible stages of the market	(P)Proba-bility	ATB (I)			OLT (j)			$(Ri - \overline{Ri}) * (Rj - \overline{Rj}) \cdot p$
		Ri	$(Ri - \overline{Ri})$	$(Ri - \overline{Ri})^2 \cdot p$	Rj	$(Rj - \overline{Rj})$	$(Rj - \overline{Rj})^2 \cdot p$	
Growth	0,5	11,55	4,32	9,33	8,58	1,95	1,90	4,21
Stagnation	0,2	6,17	-1,06	0,22	4	-2,63	1,38	0,55
Recession	0,3	0,79	-6,44	12,44	-1,58	-8,21	20,22	15,86
Total	1	x	x	$\sigma^2 = 21,99$ $\sigma = 4,69$	x	x	$\sigma^2 = 23,50$ $\sigma = 4,84$	$Cov_{ij} = 20,62$

Source: (Vlad, 2015)

$$\overline{R}_{ATB} = 11,55 \times 0,50 + 6,17 \times 0,20 + 0,79 \times 0,30 = 7,23\%$$
$$\overline{R}_{OLT} = 8,58 \times 0,50 + 4 \times 0,20 + (-1,58) \times 0,30 = 6,63\%$$

To calculate the return portfolio and the risk is necessary to determine the optimal weight of each security:

$$X^*_{ATB} = \frac{\sigma_j^2 - cov_{ij}}{\sigma_i^2 + \sigma_j^2 - 2cov_{ij}} = \frac{23,50 - 20,62}{21,99 + 23,50 - 2 \times 20,62} = 67,76\%$$

The other OLT security bond will have a weight of 100 to 67.76 = 32.24%.

$$E_{(Rp)} = 7,23 \times 0,6776 + 6,63 \times 0,3224 = 7,03\%$$
$$\sigma^2_{(p)} = (0,6776)^2 \times 21,99 + (0,3224)^2 \times 23,50 + 2 \times 0,6776 \times 0,3224 \times 20,62 = 21,54$$
$$\rho_{ij} = \frac{cov_{ij}}{\sigma_i \times \sigma_j} = \frac{20,62}{4,69 \times 4,84} = 0,90$$

To understand the changes induced in the portfolio structure by different assessments regarding market developments we must compare the obtained results with real previously recorded ones.

Table no. 2. Comparison of the portfolios

Portfolio	Securities weight		Rp	σ^2	ρ_{ij}
	ATB	OLT			
Real	55%	45%	5,19%	24,26	0,63
Risky	67,76%	32,24	7,03%	21,54	0,90

Source: (Vlad, 2015)

Improving market conditions, as predictions, is reflected in the substantial increase of expected return from 5.19% to 7.03% while there is a reduction of 2.72% of the portfolio risk. The positive influence of the predictions has manifested on return and risk by changing the weight of the two securities in favour of ATB shares, which on the optimum portfolio level is of 67.76%. Increase of the intensity correlation from 0.63 to 0.90 gives a warning about the risk of the portfolio. The proximity with the strictly positive correlation ($p_{ij} = 1$) characterizes the risk portfolio as it contains, to a very large extent, dependent on each other securities. Changing the intensity of the

correlation between securities question whether it is necessary to maintain in the portfolio both securities or only one of them. The return per unit of assumed risk and the expected return deviation can give relevant answer.

$\frac{R}{\sigma^2}(p) = 0,326; \qquad \sigma_p = 4,64$

$\frac{R}{\sigma^2}(ATB) = 0,328; \qquad \sigma_{(ATB)} = 4,69$

$\frac{R}{\sigma^2}(OLT) = 0,282; \qquad \sigma_{(OLT)} = 4,84$

Portfolio's expected return and standard deviation are very close to those of ATB security, practically creating indifference point for risk-seekers: ATB portfolio or security.

3. Risk-averse portfolio

The selection criteria took into account the association of ATB security bond, the most profitable in the market, with a ALR security bond characterized by negative market correlation and volatility (ρ = - 54; β = - 0.66). Such securities highly valued by the risk-adverse investors because they have the greatest effect for the decrease of the risk in an optimal combination.

Table no. 3. The calculation of the dispersion and covariance

The possible stages of the market	(P)Probability	ATB (I)			ALR (j)			
		Ri	$(Ri - \bar{Ri})$	$(Ri - \bar{Ri})^2 \cdot p$	Rj	$(Rj - \bar{Rj})$	$(Rj - \bar{Rj})^2 \cdot p$	$(Ri - \bar{Ri})$ $(Rj - \bar{Rj}) \cdot p$
Growth	0,5	11,55	4,32	9,33	-4,89	-4,70	11,04	- 10,15
Stagnation	0,2	6,17	- 1,06	0,22	0,98	1,17	0,27	- 0,25
Recession	0,3	0,79	- 6,44	12,44	6,85	7,04	14,87	- 13,60
Total	1	x	x	σ^2 = 21,99 σ = 4,69	x	x	σ^2 = 26,18 σ = 5,12	Cov_{ij} = -24

Source: (Vlad, 2015)

$\bar{R}_{ATB} = 7,23\% \qquad \bar{R}_{ALR} = -0,19\%$

Due to the two securities opposite evolution regarding the market, their covariance is negative.

$$X^*_{ATB} = \frac{\sigma_j^2 - cov_{ij}}{\sigma_i^2 + \sigma_j^2 - 2cov_{ij}} = \frac{26,18 - (-24)}{21,99 + 26,18 - 2(-24)} = 52,18\%$$

The ALR security will have the weight 100-52.18 = 47.82%.

$E_{(Rp)} = 0,5218 \times 7,23 + 0,4782\ (0,19) = 3,68\%$

$$\sigma^2_{(p)} = (0,5218)^2 \times 21,99 + (0,4782)^2 \times 26,18 + 2 \times 0,5218 \times 0,4782(-24) = 0$$

$$\rho_{ij} = \frac{cov_{ij}}{\sigma_i \times \sigma_j} = \frac{-24}{4,69 \times 5,12} = \frac{-24}{24} = -1$$

The comparison with previously recorded real portfolio gives the following results:

Table no. 4. Comparison of the portfolios

Portfolio	Securities weight		Rp	σ^2	ρ_{ij}
	ATB	ALR			
Real	53	47	3,73	7,25	-0,54
Risk aversion	52,18	47,82	3,68	0	-1

Source: (Vlad, 2015)

Forecasting the improving of the economic environment has not led to significant changes in the weight of the securities within the optimal composition of the portfolio. Diminishing the weight of the title with the highest return (ATB) of only 0.82% and increasing the security by the same percentage with a negative correlation (ALR) has resulted in:

- eliminating the portfolio risk by sacrificing only 0.05% of the expected return;
- achieving a strictly negative correlation between the two securities.

We have to note that such correlation is very rare in the market. The interpretation of this correlation is that 1% increase of the ATB bond return is accompanied by the same percentage decrease in the return of ALR bond and vice versa. Applying again the logic of the portfolio risk we have to see whether it is advisable to maintain in the portfolio both securities or only one of them.

$$\frac{R}{\sigma^2}(p) = 3,68; \qquad \sigma = 0 \quad \frac{R}{\sigma^2}(ATB) = 0,328; \qquad \sigma_{(ATB)} = 4,69$$

$$\frac{R}{\sigma^2}(ALR) = -0,007; \qquad \sigma_{(ALR)} = 5,12$$

From the manner in which data is presented results that for risk-adverse investor there is no question of eliminating a security; the elimination of ALR security would lead to excessive risk-taking characteristic for ATB security; ATB elimination would mean giving up a minimum of return, which again is not possible. We have to retake two if the previous observations:

- negative correlations are rare, the market being dominated by positive correlations of different intensities. In this case no correlation can be considered strictly negative, simple change of the calculation methodology with 2, 3 or 4 decimal would determine deviations.
- being risk-adverse does not mean to completely eliminate the risk, which is taken up to a certain level, usually in conjunction with the return.

4. Variation of the return and the risk in relation to the weight of securities

Market may require a level of risk below or above the limit accepted by the investor, regardless of the possible combinations. In the first case one can assume a higher risk if the return justifies the optimum portfolio, and in the second case, the investment of capital is aborted. From the data inserted into the table results different influences of the amendment of the securities over the return and risk; the return increases more if more ATB securities are introduced, while the risk decreases up to the ratio 60/40, which is also near to the optimal level of the portfolio, for which the risk is zero, then beginning to rise. The maximum return per risk unit corresponds all the same weights of 60/40. Return variation is constant for each change in composition of the portfolio of 1.48 while the risk variation is nonlinear and negative to the same ration of 60/40, then begin to grow exponentially. Comparing the portfolio which has the composition 60/40 with the optimum one (52.18 / 47.82) previously determined, it results that taking a minimal risk (0.584) leads to the increase of the unitary return with 3.61 (7.297 - 3.680), almost doubling it. Between the two extremes of the table (portfolio consisting of one title) can be made an infinite number of combinations having as landmarks the level of risk the investor is willing to assume and the return per unit of risk. At the same time we have demonstrated the earlier assertion that the weight is one of the most important factors determining the return and the risk of the portfolio

Table no. 5. Variation of the return and the risk in relation to the weight of securities

Weight (%) ATB/ALR	$E_{(Rp)}$	σ_p^2	$\frac{E_{(Rp)}}{\sigma_p^2}$	ΔE	$\Delta\sigma^2$	$\frac{\Delta E_{(Rp)}}{\Delta\sigma_p^2}$	
0/100	-0,190	26,180	-0,070	-	-	-	
20/80	1,294	9,954	0,129	1,484	-16,226	-0,091	Minimum risk
40/60	2,776	1,422	1,952	1,482	-8,532	-0,173	Maximum of return on the risk unit
60/40	4,262	0,584	7,297	1,486	-0,874	-1,700	
80/20	5,746	7,440	0,772	1,484	+6,856	0,216	
100/0	7,230	21,990	0,328	1,484	+14,550	0,102	

Source: (Vlad, 2015)

5. Conclusions

The applied methodology in handling the attitude towards risk considered in the first stage the establishment of the extreme limits, respectively the analysis of risk actions portofolio and another one with risk aversion. Later on, there was performed a simulation of the securities share mentioned in the portofolio in order to determine the optim minimal risk combination – maximum profitability for each risk unit. In the first case, the portofolio of risk actions, there was noted that the average profitability obtainedis supraunitary, but during a high volatility whichsignals taking a high risk. There is also the high addiction to market conditions. If we speak of risk aversionthere is noted a diminuishing level of the concomitrent risk eith the same tendecy of the profitability. The share securirities in a portofolio was proved to be extremely important in establishing the maximum profitability on the accepted risk unit. The adopted position at a certain moment by the investor is mosty influenced by his foreseeing ability regarding the evolution of the economic environment and in the field of activity of the company. The financial crisis presented in foreground another type of risk, the systemic one which mostly means that the insolvency of a company/financial institution determines the falling as a chain reaction of all business partners

6. References

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