

## Competition versus Monopolistic Competition by Integrating Solow-Uzawa and Dixit-Stiglitz

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### Abstract

*This paper builds a neoclassical growth model with monopolistic competition and perfect competition. The paper synthesizes the basic economic mechanisms in neoclassical growth theory and monopolistic competition within a compact framework. The economic structure is based on the Solow-Uzawa growth models. We model monopolistic competition on basis of the Dixit-Stiglitz model. This paper makes a unique contribution to growth theory by making deviations from the traditional approaches in household behavior in monopolistic competition literature and markets in neoclassical growth theory. We develop and then show behavior of the model by simulation. The calibration identifies a stable unique equilibrium point. The motion of the economy is plotted. We also studies comparative dynamic processes due to changes in degree of specialization, unit labor requirement for production of intermediates, output elasticity of intermediate inputs, propensity to save, propensity to consume service, and human capital.*

**Key words:** Dixit-Stiglitz model, monopolistic competition, Uzawa model, profit distribution, perfect competition

**J.E.L. classification:** O41; L12.

### 1. Introduction

Modeling behavior of households and market structures is the most fundamental tasks in economic theory. As far as endogenous wealth and capital dynamics is concerned, neoclassical growth theory is a main tool for economists to deal with growth on a rigor basis. The Ramsey approach to household behavior is the key tool for modelling capital and wealth accumulation in the last few decades in the theoretical literature. The Ramsey approach is not only problematical with regards to behavior mechanisms, but also causes many analytical difficulties even with simple economic questions. This may be a main reason for economists to fail to properly deal with many basic economic issues within an integrated framework. Zhang (2005, 2008) proposes an alternative approach to the Ramsey approach in order to model behavior of household with endogenous wealth. This paper applies this approach to model behavior of household to solve issues related to neoclassical growth theory with imperfect competition.

Neoclassical growth theory is developed mainly for markets of perfect competition. Modern empirical and theoretical studies in microeconomics show that most markets are characterized by imperfect competition. There is an extensive literature on imperfect competition. These studies treat market structures more realistically than neoclassical growth theory. But many important insights into functions of markets are not further integrated with neoclassical growth theory. This study attempts to make a change by introducing monopolistic competition to neoclassical growth theory. We are mainly working the core two models of neoclassical growth model, Solow's one sector growth model and Uzawa's two sector growth model. (e.g., Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995; and Ben-David and Loewy, 2003). We base our modelling economic structure on the Solow-Uzawa model (Solow, 1956; and Uzawa, 1961, see also, Stiglitz, 1967; Jensen, 2003). We assume that the consumer goods sector in the standard Uzawa model is characterized by monopolistic competition. In monopolistic competition there are numerous producers producing differentiated products. Products are

differentiated. No perfect is a perfect substitute any other. Each firm takes the prices charged by other firms as given and maximizes its profit. Each firm has some degree of market power. Market power is measured by power over the demand and supply equilibrium terms and conditions. Formal monopolistic competition theory was developed first by Chamberlin (1933). The theory has since then been developed by many scholars. In the last two decades it has been developed in numerous ways in analyzing economic growth and development, economic structures, innovation and technological diffusion, and economic geography (e.g., Dixit and Stiglitz, 1977; Krugman, 1979; Ethier, 1982; Romer, 1990; Brakman and Heijdra, 2004; Behrens and Murata, 2007, 2009; and Wang, 2012). Zhang (2018) makes an alternative attempt to integrate monopolistic competition theory and neoclassical growth theory on basis of the Solow one sector model and Dixit-Stiglitz model (Dixit-Stiglitz, 1977; see also Lancaster, 1980; Waterson, 1984; Benassy, 1996; Picard and Toulemonde, 2009; and Parenti, *et.al.*, 2017). The study extends Zhang's model by adding the consumer goods sector. Rather than intermediate goods being used as consumer goods as in the Dixit-Stiglitz model, intermediate goods are used as inputs of the final goods sector as in the Grossman-Helpman model (Grossman and Helpman, 1990). We differ from the other studies in that the profits of intermediate inputs sectors are shared equally among the homogeneous population, but not for investment. We organize the rest of the paper as follows. Section 2 gives the three-sector growth model. Section 3 makes an analysis of the model and simulates the movement of the dynamic model. Section 4 makes comparative dynamic analysis in some parameters. Section 5 concludes the study.

## 2. Theoretical background - The growth model with intermediate inputs

We synthesize the three important models (Solow, 1956; Uzawa, 1961; and Dixit and Stiglitz, 1977) to give a dynamic general equilibrium analysis of economic growth with perfect competition and monopolistic competition. There are three kinds of production for supply a final (capital) good (like in the Solow model), consumer goods and services (like in the Uzawa model), and a variety of differentiated middle products (like in the Dixit-Stiglitz model). Different from the Grossman-Helpman approach and the which neglect wealth and capital accumulation, we follow the neoclassical grow theory in considering capital accumulation as the main machine of economic growth. The three sectors fully employs the labor force.

### *The Capital Goods Sector*

Let  $K_i(t)$ ,  $N_i(t)$ , and  $F_i(t)$  denote capital input, labor input, and output of the final goods sector, respectively. Capital good is used numeraire. Physical capital depreciates at a given rate  $\delta_k$ . We use  $n$  to represent the number of varieties of middle products available. A rise in  $n$  means an increase in the degree of specialization. We use  $X_i(t)$  to represent aggregate input of intermediate inputs as follows:

$$X_i(t) = \sum_{\varepsilon=1}^n x_{\varepsilon}^{\theta}(t), 0 < \theta < 1, \quad (1),$$

in which  $x_{\varepsilon}(t)$  represents the input of middle product  $\varepsilon$  and  $\theta$  is a parameter. We Grossman and Helpman (1990) in describing the production of final goods. We choose the following production function:

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t) X_i^{\gamma_i}(t), \quad 0 < \alpha_i, \beta_i, \alpha_i + \beta_i < 1, \quad \gamma_i = \frac{1 - \alpha_i - \beta_i}{\theta} < 1,$$

in which  $A_i$ ,  $\alpha_i$  and  $\beta_i$  stand for parameters. The specified form implies constant returns to scale for given  $n$ , but increasing in  $n$ . It also implies an improvement in technical efficiency due to an increasing degree of specialization. Scale economies exist at the industry level. For individual firms, scale economies are exogenous.

Variables  $r(t)$ ,  $w(t)$ , and  $p_{\varepsilon}(t)$  represent, respectively, the rate of interest, the wage rate, and the price of middle good  $\varepsilon$ . We have the profit of the capital goods sector:

$$\pi_i(t) = F_i(t) - (r(t) + \delta_k)K_i(t) - w(t)N_i(t) - \sum_{\varepsilon=1}^n p_{\varepsilon}(t)x_{\varepsilon}(t). \quad (2)$$

The marginal conditions are given by:

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \quad p_\varepsilon(t) = \frac{\gamma_i \theta x_\varepsilon^{\theta-1}(t) F_i(t)}{X_i(t)}. \quad (3)$$

The production function formation tells that the share of factor  $X_i$  equals  $\gamma_i F_i$ . Equations (2) and (3) imply the following relations:

$$K_i(t) = \Lambda(t) X_i(t), \quad N_i(t) = \left( \frac{w(t)}{\beta_i A_i K_i^{\alpha_i}(t) X_i^{\gamma_i}(t)} \right)^{1/(\beta_i-1)}, \quad (4)$$

$$\Lambda(r, w, t) \equiv \left[ \left( \frac{r_\delta(t)}{\alpha_i A_i} \right)^{\beta_i-1} \left( \frac{\beta_i A_i}{w(t)} \right)^{\beta_i} \right]^{\frac{1}{\gamma_i}}, \quad r_\delta(t) \equiv r(t) + \delta_k.$$

We see that  $\Lambda(t)$  is independent of variety. From (3), we get:

$$p_\varepsilon(t) = \frac{\gamma_i \theta_i r_\delta(t) x_\varepsilon^{\theta-1}(t) K_i(t)}{\alpha_i X_i(t)}. \quad (5)$$

Inserting (4) in (5) implies:

$$x_\varepsilon(t) = \tilde{\Lambda}(t) p_\varepsilon^{-\bar{\theta}}(t), \quad (6)$$

in which

$$\tilde{\Lambda}(t) \equiv \left( \frac{\gamma_i \theta_i r_\delta(t) \Lambda(t)}{\alpha_i} \right)^{\bar{\theta}}, \quad \bar{\theta} \equiv \frac{1}{1-\theta}.$$

We see that  $\tilde{\Lambda}(t)$  is independent of variety. The share of variety  $\varepsilon$  in the total value of intermediate inputs is:

$$\varphi_\varepsilon(t) \equiv \frac{x_\varepsilon(t) p_\varepsilon(t)}{\sum_{m=1}^n x_m(t) p_m(t)}. \quad (7)$$

Insert (6) in (7)

$$\varphi_\varepsilon(t) = \frac{p_\varepsilon^{1-\bar{\theta}}(t)}{\sum_{m=1}^n p_m^{1-\bar{\theta}}(t)}. \quad (8)$$

The share variety  $\varepsilon$  is a function of its price and the aggregated price.

### The Middle Goods Sector

Following Dixit and Stiglitz (1977) we describe production of middle goods. It is assumed that the production is oligopolistic with price competition. The producer of variety  $\varepsilon$  gets the profit:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_N w(t)] \frac{\varphi_\varepsilon(t) \gamma_i F_i(t)}{p_\varepsilon(t)},$$

where  $a_N$  is the labor requirement for producing unit of the intermediate. It expresses that the profit equals the profit from per unit of product multiplied by the share of the market. Inserting (8) in the above profit yields:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_N w(t)] \frac{\gamma_i F_i(t) p_\varepsilon^{-\bar{\theta}}(t)}{\sum_{m=1}^n p_m^{1-\bar{\theta}}(t)}, \quad (9)$$

The producer decides  $p_\varepsilon(t)$  in order to get maximal profit. From (3) and (1), we have

$$F_i(t) = \frac{p_\varepsilon(t)}{\gamma_i \theta x_\varepsilon^{\theta-1}(t)} \sum_{\varepsilon=1}^n x_\varepsilon^\theta(t). \quad (10)$$

Insert (6) in (10)

$$\frac{F_i(t)}{\sum_{m=1}^n p_m^{1-\bar{\theta}}(t)} = \frac{\tilde{\Lambda}(t)}{\gamma_i \theta}. \quad (11)$$

With (9) and (11), we now express the profit function:

$$\pi_\varepsilon(t) = [p_\varepsilon(t) - a_N w(t)] \frac{\gamma_i \tilde{\Lambda}(t) p_\varepsilon^{-\bar{\theta}}(t)}{\gamma_i \theta}.$$

The first-order condition (i.e.,  $\partial \pi_\varepsilon / \partial p_\varepsilon = 0$ ) yields the fixed-markup pricing rule:

$$\theta p_\varepsilon(t) = a_N w(t). \quad (12)$$

This equation also implies that varieties have the same price. With (9) and (12), we have the profit per firm:

$$\pi(t) = \frac{(1 - \theta)\gamma_i F_i(t)}{n}. \quad (13)$$

We see that the profit per firm is independent of  $\varepsilon$ . From (5), we see  $x_\varepsilon(t)$  being independent of  $\varepsilon$ . We express this variable by  $x(t)$ . From (1) we get:

$$X_i(t) = nx^{\theta_i}(t). \quad (14)$$

Total profit is:

$$\bar{\pi}(t) = n\pi(t). \quad (15)$$

The industry's total profit equals the industry's number of the firms by the representative firm's profit

### **The Service Sector**

We denote capital input, labor input, output of the service sector, respectively by  $K_s(t)$ ,  $N_s(t)$ , and  $F_s(t)$ . We use  $p_s(t)$  to stand for price of services. We specify a production function of services as follows:

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \quad 0 < \alpha_s, \beta_s < 1, \quad \alpha_s + \beta_s = 1, \quad (16)$$

in which  $A_s$ ,  $\alpha_s$  and  $\beta_s$  are fixed coefficients. The first-order conditions yield:

$$r(t) + \delta_k = \frac{\alpha_s p_s(t) F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s p_s(t) F_s(t)}{N_s(t)}. \quad (17)$$

We use (17) to determine the distribution of labor and capital inputs.

### **Consumer Behaviors and Wealth Dynamics**

This study applies the model of consumer behavior suggested by Zhang (1993, 2005). We use  $\bar{k}(t)$  to represent per household wealth. We have  $\bar{k}(t) = K(t)/N$ , where  $K(t)$  represents the total capital stock. We assume that total profits are equally shared among households. There are different ways that profits are distributed. For instance, in the new growth theory profits are invested for innovation. For simplicity, this study assumes profit to be shared equally between the homogenous households. We take a simplified approach at this initial stage of modeling. Let  $h$  stand for human capital. The household's current income is:

$$y(t) = r(t)\bar{k}(t) + hw(t) + \frac{\bar{\pi}(t)}{N}. \quad (18)$$

The disposable income  $\hat{y}(t)$  is defined, different from the current term used in the literature, as the sum of the household's wealth value and current disposable income:

$$\hat{y}(t) = y(t) + \bar{k}(t) = (1 + r(t))\bar{k}(t) + hw(t) + \frac{\bar{\pi}(t)}{N}. \quad (19)$$

The disposable income is spent entirely on consuming goods  $c_i(t)$ , services  $c_s(t)$ , and saving  $s(t)$ . We have the budget constraint as follows:

$$c_i(t) + p_s(t)c_s(t) + s(t) = \hat{y}(t). \quad (20)$$

The household makes decision on consumption levels of goods and services and amount of saving. The household's utility level  $U(t)$  is related to  $c_i(t)$ ,  $c_s(t)$  and  $s(t)$  as follows:

$$U(t) = c_i^{\xi_0}(t) c_s^{\chi_0}(t) s^{\lambda_0}(t), \quad \xi_0, \chi_0, \lambda_0 > 0,$$

where we use  $\chi_0$  to represent the propensity to consume services,  $\xi_0$  the propensity to consume capital goods, and  $\lambda_0$  the propensity to save. Maximize the utility s.t.: (20)

$$c_i(t) = \xi \hat{y}(t), \quad p_s(t)c_s(t) = \chi \hat{y}(t), \quad s(t) = \lambda \hat{y}(t), \quad (21)$$

$$\rho \equiv \frac{1}{\xi_{0+} + \chi_0 + \lambda_0}, \quad \chi \equiv \rho \chi_0, \quad \xi \equiv \rho \xi_0, \quad \lambda \equiv \rho \lambda_0.$$

We express the change in the wealth as net saving which implies saving minus dissaving:

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t). \quad (22)$$

### **Equilibrium Conditions for Final goods and Services**

Changes in the capital stock are the final goods sector's output minus the consumption of capital goods and the depreciated amount of capital stock. We have:

$$\dot{K}(t) = F_i(t) - c_i(t)N - \delta_k K(t). \quad (23)$$

Equilibrium condition for the service sector implies:

$$c_s(t)N = F_s(t). \quad (24)$$

### **Full Employment of Labor and Capital**

The labor force of intermediate goods sector is

$$N_x(t) = a_N x(t)n.$$

We have the balance condition for labor as follows:

$$N_i(t) + N_s(t) + a_N x(t)n = hN, \quad (25)$$

The balance condition for capital is as follows:

$$K_i(t) + K_s(t) = K(t). \quad (26)$$

The left-hand side is the sum of the capital stocks employed by the capital goods and consumer goods sector and the right-hand is the total physical capital available in the economy.

### **National Capital and National Wealth**

The value of national wealth is given as the value of physical capital:

$$\bar{k}(t)N = K(t). \quad (27)$$

The model is based on the Solow-Uzawa model, the Dixit-Stiglitz model, and the Grossman-Helpman model. These models are integrated with Zhang's utility function and concept of disposable income. We now examine the model's properties.

## **3. The model's dynamic properties**

The previous section developed a neoclassical growth model, integrating a few core models in the literature of growth theory. As we are mainly concerned with simulation by computer, we provide a computational program for simulating the movement of the economic dynamics.

### **Lemma**

The following differential equation determines the motion of the economic system:

$$\dot{x}(t) = \left( \frac{dv(x(t))}{dx} \right)^{-1} f(x(t)), \quad (28)$$

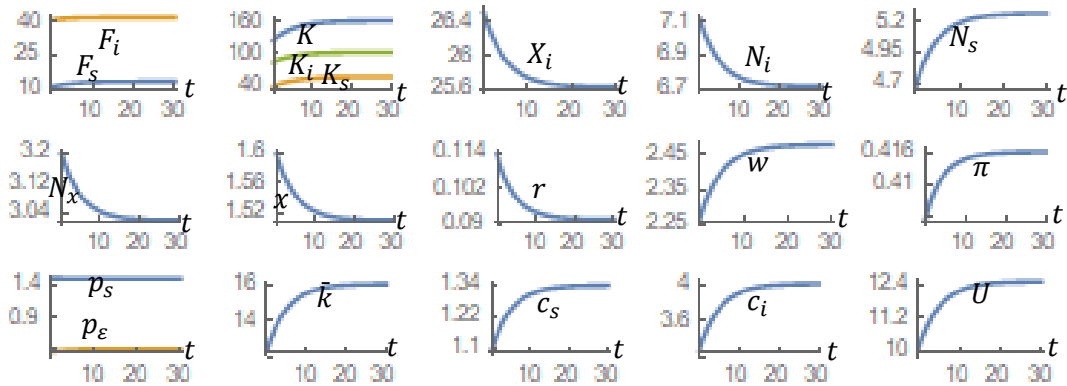
where the Appendix gives functions  $v(x(t))$  and  $f(x(t))$ . All the other variables are computed with the following procedure as functions of  $x(t)$ :  $\bar{k}(t)$  by (A12)  $\rightarrow K(t) = \bar{k}(t)N \rightarrow p_\varepsilon(t)$  with (A2)  $\rightarrow F_i(t)$  with (A8)  $\rightarrow N_i(t)$  from (A4)  $\rightarrow N_s(t)$  from (A5)  $\rightarrow K_i(t)$  and  $K_s(t)$  from (A1)  $\rightarrow F_s(t)$  with (16)  $\rightarrow w(t)$  with (A9)  $\rightarrow p_s(t)$  from (17)  $\rightarrow \hat{y}(t)$  from (A10)  $\rightarrow r(t)$  with (A13)  $\rightarrow \varphi(t)$  with (8)  $\rightarrow c_i(t)$ ,  $c_s(t)$  and  $s(t)$  from (21)  $\rightarrow \pi(t)$  from (13)  $\rightarrow \bar{\pi}(t)$  with (15)  $\rightarrow X_i(t)$  with (14).

The expressions are complicated. It is not easy to give simple intuitive interpretations of the results. We illustrate dynamic behavior of the model by simulation. The parameters are taken on the following values:

$$n = 20, N = 10, \theta = 0.6, a_N = 0.4, A_i = 1.4, A_s = 1.1, \alpha_i = 0.3, \beta_i = 0.4, \\ \alpha_s = 0.35, \lambda_0 = 0.8, \xi_0 = 0.2, \chi_0 = 0.1, h = 1.5, \delta_k = 0.03. \quad (29)$$

We fix the population at 10 and the level of human capital at 1.5. We specify the number of varieties of intermediate inputs 20. We choose the initial condition:  $x(0) = 1.6$ . Figure 1 shows the simulation result.

Figure no. 1. The Motion of the Model



Source: Author's computation

The output levels of the capital good and service sectors are augmented from the initial state. The two sectors' capital inputs are enhanced. National wealth rises. The labor inputs employed by the intermediate goods sector and capital good sector fall. The service sector uses more labor force. There is almost no change in the prices. The rate of interest decreases. The wage rate is enhanced. The household's consumption levels of services and capital good are increased. The household enjoys higher utility. The sector uses more net input of intermediate goods. The economy produces less output of each variety. Each firm in the intermediate goods sector earns more profit. The equilibrium values of the variables are calculated as follows:

$$\begin{aligned}
 K &= 160.6, F_i = 41.6, F_s = 13.4, X_i = 25.6, N_i = 6.7, N_s = 5.3, N_x = 3, \\
 K_i &= 102.8, K_s = 57.8, x = 1.51, \pi = 0.42, r = 0.09, \\
 w &= 2.48, p_s = 1.5, p_\epsilon = 0.41, \bar{k} = 16.1, c_i = 4, c_s = 1.34, U = 12.5.
 \end{aligned}$$

#### 4. Transitory and long-term effects in change of parameters

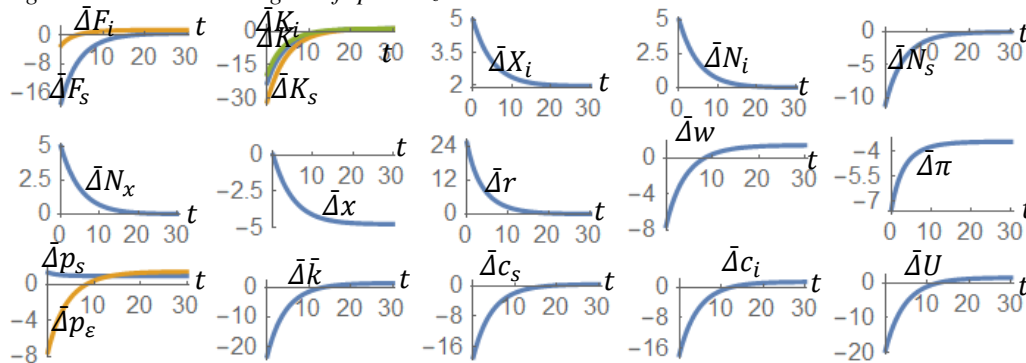
The previous section simulated the dynamics of the model. The next step is to ask about effects of exogenous changes in parameter values on transitory processes and long-term values of the variables. To do this, we define a symbol  $\hat{\Delta}x_j(t)$  to express the change rate of  $x_j(t)$  in percentage due to the exogenous changes.

##### 4.1. A rise in degree of specialization

We first deal with the effects on the economic system when the degree of specialization is increased in the following way:  $n = 20 \Rightarrow 21$ . Figure 2 shows the simulation result. We have initially rises in the the output levels of the capital good and service sectors, then decreases in the same variables, and finally rises in the long term. The two sectors' employment of capital input is initially increased and is almost invariant in the long term. The economy experiences initial falls in the national wealth (total capital stock) and long-term rises. The employment of labor inputs by the intermediate goods sector and capital good sector rise initially and remain almost invariant in the long term. The employment of labor input by the service sector rises initially and reminds almost invariant in the long term. The price of service rises. The economy experiences an initial fall in the price of intermediate goods but long-term rise. The economy has an initial rise in the rate of interest but reminds invariant in the long term. The wage rate is reduced initially but augmented in the long term. The household's level of wealth falls lower initially but slightly higher in the long term. The household's consumption of services falls. The consumption level of capital good is reduced initially and almost the same in the long term. The household has lower utility initially but higher

in the long term. The economy experiences an expansion in the net input of intermediate goods. The economy produces less each variety. The firm's profit falls.

Figure no. 2. A rise in degree of specialization

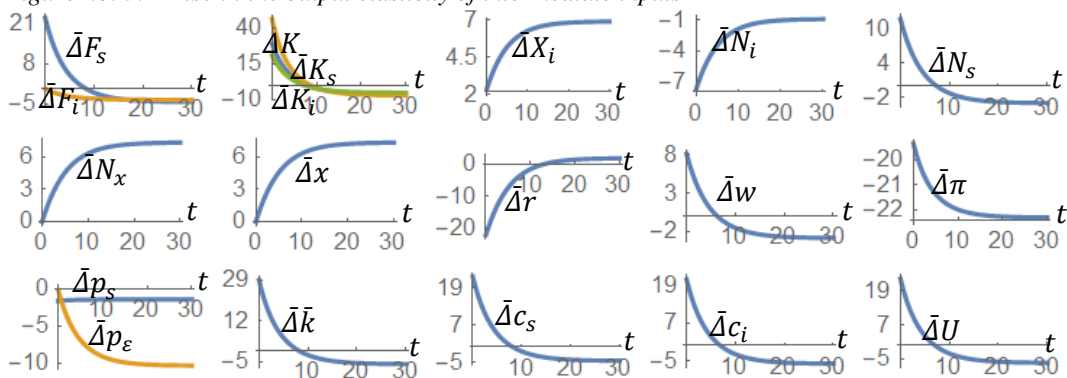


Source: Author's computation

#### 4.2. A rise in the output elasticity of intermediate inputs

We now deal with the effects of the following increase in the output elasticity of intermediate inputs on the system:  $\theta = 0.6 \Rightarrow 0.65$ . Figure 3 shows the simulation result. The service sector's output level is increased initially but reduced in the long term. The capital good sector's output level is decreased. The capital inputs employed by the two sectors are augmented initially but reduced in the long term. The household has more national wealth initially but less in the long term. The labor force employed by the intermediate goods sector rises. The labor force employed by the capital good sector falls. The labor employment by the service sector rises initially but falls in the long term. The economy has lower price of service and intermediate goods. The rate of interest is decreased initially but increased in the long term. The wage rate rises initially but falls in the long term. The wealth is increased initially but reduced in the long term. The household's consumption of services and capital good rise initially but fall in the long term. The household has higher utility initially but lower in the long term. The net input of intermediate goods rises. Each firm's output is increased in association with falling profit.

Figure no. 3. A rise in the output elasticity of intermediate inputs



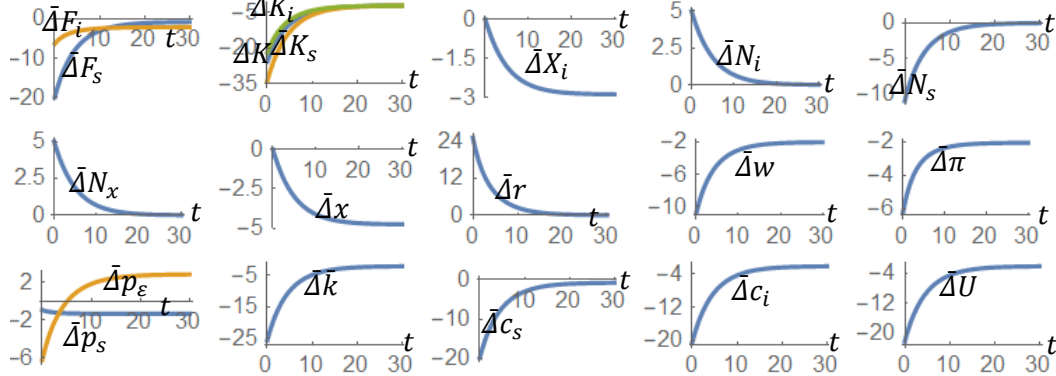
Source: Author's computation

#### 4.3. A rise in the unit labor requirement in producing intermediates

We now analyze the effects that the following rise in the unit labor requirement in producing intermediates:  $a_N = 0.1 \Rightarrow 0.105$ . Figure 4 shows the simulation result. The capital good and service sectors' output levels are reduced. The capital inputs employed by the two sectors fall. The economy has lower national wealth. The labor inputs employed by the intermediate goods sector

and capital good sector fall initially and remind almost invariant in the long term. The labor input employed by the service sector falls initially and reminds almost invariant in the long term. The price of service is decreased. The price of intermediate goods is decreased initially but increased in the long term. The rate of interest is increased initially and reminds almost invariant in the long term. The wage rate is reduced. The household's wealth is reduced. The consumption levels of services and capital good fall. The household has lower utility level. The net input of intermediate goods fall. Each firm in the intermediate goods sector produces less and has less profit.

Figure no. 4. A rise in the unit labor requirement in producing intermediates

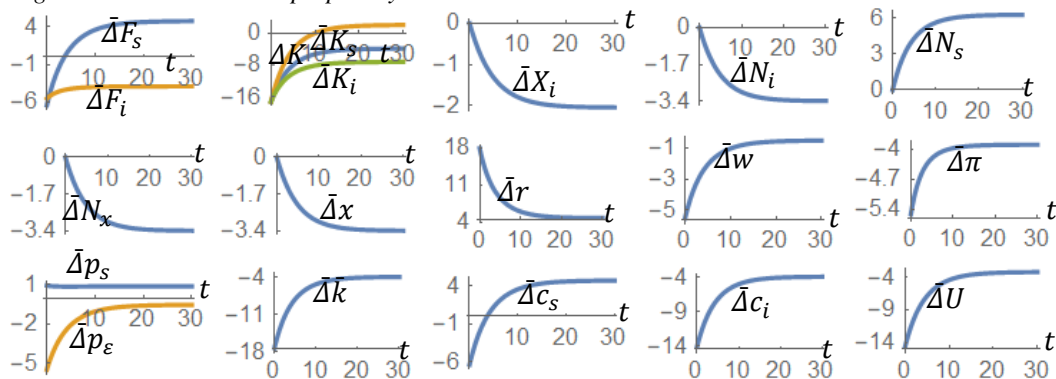


Source: Author's computation

#### 4.4. A rise the propensity to consume service

We now study the effects of the following rise in the propensity to consume services on the economy:  $\chi_0 = 0.4 \Rightarrow 0.42$ . Figure 5 shows the simulation result. The capital good sector produces. The service sector produces less initially but more in the long term. The capital input by the capital good sector falls. The capital input employed by the service sector falls initially but rises in the long term. The economy has less capital. The labor inputs employed by the capital good sector and intermediate goods sector are reduced. The labor input employed by the service sector is augmented. The price of service is enhanced. The price of intermediate goods is lowered. The economy experiences in the rate of interest. The wage rate is reduced. The household owns less wealth. The consumption of services falls initially but rises in the long term. The consumption of capital goods by the household falls. The household has lower utility level. The economy use less input of intermediate goods. Each firm in the intermediate goods sector produces less and has less profit.

Figure no. 5. A rise in the propensity to consume service



Source: Author's computation

#### 5. Conclusions

This study modelled growth of an economy in which has perfect competitive and monopolistic competition markets. The economic structure is constructed by synthesizing the economic mechanisms



in neoclassical growth theory and the monopolistic competition. We made a unique contribution to the literature of economic growth by integrating two main modeling frameworks with Zhang's utility function and concept of disposable income. We calibrated the model. The system has a stable unique equilibrium point. We also plotted the movement of the model. We also showed the effects of changes in some parameters on the transitory processes and long-term equilibrium values of the variables. As the model is built on the core (simplified) models of growth theory and monopolistic competition, it is not difficult to extend and generalize

We now check the Lemma. We define  $z \equiv (r + \delta_k)/w$ . With (3) we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{\bar{\beta}_i N_i}{K_i} = \frac{\bar{\beta}_s N_s}{K_s}, \quad (A1)$$

where  $\bar{\beta}_j \equiv \alpha_j/\beta_j$ ,  $j = i, s$ . From (12) we have

$$p_\varepsilon = \frac{a_N w}{\theta}. \quad (A2)$$

From (3), we have

$$w = \frac{\beta_i p_\varepsilon n x}{N_i \gamma_i \theta}, \quad (A3)$$

where we also use (14). By (A2) and (A3), we have

$$N_i = a x, \quad a \equiv \frac{\beta_i n a_N}{\gamma_i \theta^2}. \quad (A4)$$

From (A4) and (25), we have

$$N_s = hN - a_0 x, \quad (A5)$$

where  $a_0 \equiv a + a_N n$ . From (21), (17) and (24) we have

$$\hat{y} = \frac{w N_s}{\beta_s \chi N}. \quad (A6)$$

By (2) we have

$$F_i = A_i N_i \left(\frac{K_i}{N_i}\right)^{\alpha_i} \left(\frac{X_i}{N_i}\right)^{\gamma_i}. \quad (A7)$$

Insert (A1) and (A4) in (A7)

$$F_i(x, z) = A_i a x \left(\frac{\bar{\beta}_i}{z}\right)^{\alpha_i} \left(\frac{n x^{\theta-1}}{a}\right)^{\gamma_i}. \quad (A8)$$

From (3), (A8) and (A4), we have

$$w(x, z) = \frac{\beta_i F_i(x, z)}{a x}. \quad (A9)$$

With (A6) and (A5), we obtain

$$\hat{y} = \frac{(hN - a_0 x) w}{\beta_s \chi N}. \quad (A10)$$

From (26), (27) and (A1), we solve

$$\bar{\beta}_i N_i + \bar{\beta}_s N_s = z \bar{k} N. \quad (A11)$$

Insert (A4) and (A5) in (A11)

$$\bar{k} = \frac{a_1 x}{z N} + \frac{\bar{\beta}_s h}{z}, \quad (A12)$$

where  $a_1 \equiv \bar{\beta}_i a - a_0 \bar{\beta}_s$ . From (A1) we have

$$r(x, z) = w z - \delta_k. \quad (A13)$$

From (A10) and (19), we have

$$h w + \frac{(1 - \theta) \gamma_i F_i}{N} + (1 + r) \bar{k} - \frac{(hN - a_0 x) w}{\beta_s \chi N} = 0, \quad (A14)$$

where we also use (13) and (15). Insert (A13) and (A12) in (A14)

$$\Lambda_1 w z - \frac{(1 - \theta) \gamma_i z F_i}{N} = \Lambda_0, \quad (A15)$$

where  $\delta \equiv 1 - \delta_k$  and

$$\Lambda_1(x) \equiv \frac{hN - n_0x}{\beta_s \chi N} - \frac{a_1 x}{N} - \bar{\beta}_s h - h, \quad \Lambda_0(x) \equiv \frac{a_1 \delta x}{N} + \delta \bar{\beta}_s h.$$

From (A9), (A16) and (A8) we solve

$$z = \omega(x) \equiv \left[ \frac{\Lambda_0}{A_i a x \bar{\beta}_i^{\alpha_i}} \left( \frac{\beta_i \Lambda_1}{a x} - \frac{(1-\theta)\gamma_i}{N} \right)^{-1} \left( \frac{a}{n x^{\theta-1}} \right)^{\gamma_i} \right]^{1/(1-\alpha_i)}. \quad (A16)$$

We get the computational procedure in the Lemma. From the procedure and (22), (A11) and (A5), we get

$$\dot{\bar{k}} = f(x) \equiv s - \bar{k}. \quad (A17)$$

Denote (A12) by  $\bar{k} = v(x)$ . We have

$$\dot{\bar{k}} = \frac{dv}{dx} \dot{x}. \quad (A18)$$

From (A18) and (A19), we have

$$\dot{x} = \left( \frac{dv}{dx} \right)^{-1} f. \quad (A19)$$

## 7. References

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