Choosing the Appropriate Regression from Nine Different Mathematical Models

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Abstract

Choosing the most appropriate mathematical model for establishing a simple regression between two variables which represent two economic indicators or from other scientific fields is an approach that involves many and often quite difficult actions.

In the article we rigorously presented the stages to be followed in order to obtain the defining parameters for nine models: linear, power (double logarithmic), exponential, logarithmic, three hyperbolic models (the third is also a Törnqvist-type model) and two exponential-hyperbolic models.

The final stage in establishing the most appropriate model is less discussed in the paper. Thus, the simple regression for which the model has the highest Pearson correlation coefficient is chosen as optimal in the article.

Key words: simple regressions, power model, exponential model, logarithmic model, hyperbolic model
J.E.L. classification: C02, C24, C40, C50

1. Introduction

An important desideratum in conducting research in the economic field, as well as in many other scientific fields, is defined by the question "which mathematical model is the most appropriate to describe the connection between two indicators"?

Selecting the optimal regression from as many mathematical models as possible will result in a viable link between the two indicators.

The main objective of the paper is to describe, as concisely as possible and also as completely as possible, the way in which the parameters that define nine different mathematical models are obtained.

Starting from the linear model, described in detail in many papers in the literature, all the other eight power, exponential, logarithmic and hyperbolic models can be linearized if appropriate logarithmic operations and variable changes are applied.

After linearization, the mathematical relations of the linear model will be used through the substitutions corresponding to each model.

Another objective of the paper is the effective selection of the model that generates the optimal simple regression.

This objective is achieved by associating the nine correlation coefficients and marking the model that has the highest coefficient.

2. Literature review

The article is based upon the linear model for achieving a simple correlation.

The complete theoretical presentation of this model can be found in many works, both nationally and internationally.

Of these, we mention here, in chronological order, just as an example, the following works: (Schatteles, 1971), (Bărbat, 1972) (Moineagu, 1974), (Ţarcă, 1979), (Trebici *et al*, 1985), (Jaba, 1986), (Gromîco, 1990), (Pecican, 1994), (Baron *et al*, 1996), (Biji *et al*, 2000), (Gherasim, 2021) and so on.

The other simple correlations used in the article are based upon power, exponential, logarithmic, hyperbolic models and combinations thereof.

In the literature these models are often only briefly mentioned and rarely presented completely, both theoretically and applied.

Also by way of example, we mention here only a few works in which curvilinear models are presented: (Schatteles, 1971, pp.127-128), (Moineagu, 1974, pp.40-43), (Ţarcă, 1979, pp.61-62), (Jaba 1986, pp.205-206), (Gromîco, 1990, pp.140-143), (Pecican, 1994, pp.64-65), (Baron *et al*, 1996, pp.168-169), (Biji *et al*, 2000, pp.314-318) and so forth.

3. Research methodology

To establish a simple regression $y = f(x)$ between an independent variable x and a dependent variable y are considered given a sufficiently large number n of value pairs of two economic-social indicators or in other fields: $(x_k, y_k)_{k=\overline{1,n}}$ $, 30 \le n \in \mathbf{N}$.

By graphically representing these pairs in a two-dimensional space, a point cloud is obtained $\{P_k(x_k, y_k)\}_{k=\overline{1,n}}$.

Throughout the article we used nine mathematical models for the type and shape of the regression function f:

$$\left.\begin{array}{llll}
(1) & y = a\cdot x + b & & \text{, linear model} \\
(2) & y = b\cdot x^a & \underset{\log}{\rightarrow} \ \hat{y} = a\cdot\hat{x} + \hat{b} & \text{, power (double logarithmic) model} \\
(3) & y = b\cdot e^{a\cdot x} & \underset{\log}{\rightarrow} \ \hat{y} = a\cdot x + \hat{b} & \text{, exponential model} \\
(4) & y = a\cdot\ln x + b & \rightarrow \ y = a\cdot\hat{x} + b \ , \ (\ln x \rightarrow \hat{x}) & \text{, logarithmic model} \\
(5) & y = \dfrac{a}{x} + b & \rightarrow \ y = a\cdot X + b \ , \ \left(\dfrac{1}{x} \rightarrow X\right) & \text{, hyperbolic model type 1} \\
(6) & y = \dfrac{1}{a\cdot x + b} & \rightarrow \ Y = a\cdot x + b \ , \ \left(\dfrac{1}{y} \rightarrow Y\right) & \text{, hyperbolic model type 2} \\
(7) & y = \dfrac{x}{b\cdot x + a} & \rightarrow \ Y = a\cdot X + b \ , \ \left(\dfrac{1}{x} \rightarrow X\right), \left(\dfrac{1}{y} \rightarrow Y\right) & \text{, hyperbolic model type 3} \\
(8) & y = b\cdot e^{\frac{a}{x}} & \underset{\log}{\rightarrow} \ \hat{y} = a\cdot X + \hat{b} \ , \ \left(\dfrac{1}{x} \rightarrow X\right) & \text{, exponential-hyperbolic model type 1} \\
(9) & y = e^{\frac{1}{a\cdot x + b}} & \underset{\log}{\rightarrow} \ Y = a\cdot x + b \ , \ \left(\dfrac{1}{\hat{y}} \rightarrow Y\right) & \text{,exponential-hyperbolic model type 2}
\end{array}\right] \quad (1)$$

where: $\ln w \overset{\text{not.}}{=} \hat{w}$ $, \forall w$.

Model (7) is also a Törnqvist model $\left(y = \dfrac{\alpha\cdot x}{x + \beta}\right)$, where: $\alpha = \dfrac{1}{b}$ and $\beta = \dfrac{a}{b}$.

Model (1) is linear and the other eight models become linear by logarithmic and variable changes presented in parentheses.

In order to determine the parameters that define the nine models, $a_{(M)}$, $b_{(M)}$ $, M = \overline{1,9}$ it is necessary to go through several stages.

In a first stage, relative to the coordinates x_k, y_k, logarithms, inverses, products and their amounts must be calculated:

$$\left.\begin{array}{l}
S_x=\sum_{k=1}^{n}x_k\,,\quad S_y=\sum_{k=1}^{n}y_k\,,\quad S_{xx}=\sum_{k=1}^{n}(x_k\cdot x_k)\,,\quad S_{xy}=\sum_{k=1}^{n}(x_k\cdot y_k)\,,\quad S_{\hat{x}}=\sum_{k=1}^{n}\ln x_k \\
S_{\hat{y}}=\sum_{k=1}^{n}\ln y_k\,,\quad S_{\hat{x}\hat{x}}=\sum_{k=1}^{n}(\ln x_k\cdot\ln x_k)\,,\quad S_{x\hat{y}}=\sum_{k=1}^{n}(x_k\cdot\ln y_k)\,,\quad S_{\hat{x}y}=\sum_{k=1}^{n}(y_k\cdot\ln x_k) \\
S_{\hat{x}\hat{y}}=\sum_{k=1}^{n}(\ln x_k\cdot\ln y_k)\,,\quad S_{\frac{1}{x}}=\sum_{k=1}^{n}\frac{1}{x_k}\,,\quad S_{\frac{1}{y}}=\sum_{k=1}^{n}\frac{1}{y_k}\,,\quad S_{\frac{y}{x}}=\sum_{k=1}^{n}\frac{y_k}{x_k}\,,\quad S_{\frac{x}{y}}=\sum_{k=1}^{n}\frac{x_k}{y_k} \\
S_{\frac{1}{xx}}=\sum_{k=1}^{n}\frac{1}{x_k\cdot x_k}\,,\quad S_{\frac{1}{xy}}=\sum_{k=1}^{n}\frac{1}{x_k\cdot y_k}\,,\quad S_{\frac{1}{\hat{y}}}=\sum_{k=1}^{n}\frac{1}{\hat{y}_k}\,,\quad S_{\frac{x}{\hat{y}}}=\sum_{k=1}^{n}\frac{x_k}{\hat{y}_k}\,,\quad S_{\frac{\hat{y}}{x}}=\sum_{k=1}^{n}\frac{\hat{y}_k}{x_k}
\end{array}\right]\quad (2)$$

The relations by which the two parameters of the linear model are calculated are described in many works of which we give as an example: (Schatteles, 1971, pp.120-126), (Ţarcă, 1979, pp.11-25), (Jaba, 1986, pp.207-209), (Gromîco, 1990, pp.135-137) and so on.

For the other eight models linearized by the actions presented above (logarithmic and variable changes) the parameters $a_{(M)}$ and $b_{(M)}$ are deduced from the relations of the linear model:

$$\left.\begin{array}{lll}
a_{(1)}=\dfrac{n\cdot S_{xy}-S_x\cdot S_y}{n\cdot S_{xx}-S_x\cdot S_x}\,, & b_{(1)}=\dfrac{S_{xx}\cdot S_y-S_x\cdot S_{xy}}{n\cdot S_{xx}-S_x\cdot S_x} & \\
\hline
a_{(2)}=\dfrac{n\cdot S_{\hat{x}\hat{y}}-S_{\hat{x}}\cdot S_{\hat{y}}}{n\cdot S_{\hat{x}\hat{x}}-S_{\hat{x}}\cdot S_{\hat{x}}}, & \hat{b}_{(2)}=\dfrac{S_{\hat{x}\hat{x}}\cdot S_{\hat{y}}-S_{\hat{x}}\cdot S_{\hat{x}\hat{y}}}{n\cdot S_{\hat{x}\hat{x}}-S_{\hat{x}}\cdot S_{\hat{x}}}, & b_{(2)}=e^{\hat{b}_{(2)}} \\
a_{(3)}=\dfrac{n\cdot S_{x\hat{y}}-S_x\cdot S_{\hat{y}}}{n\cdot S_{xx}-S_x\cdot S_x}, & \hat{b}_{(3)}=\dfrac{S_{xx}\cdot S_{\hat{y}}-S_x\cdot S_{x\hat{y}}}{n\cdot S_{xx}-S_x\cdot S_x}, & b_{(3)}=e^{\hat{b}_{(3)}} \\
a_{(4)}=\dfrac{n\cdot S_{\hat{x}y}-S_{\hat{x}}\cdot S_y}{n\cdot S_{\hat{x}\hat{x}}-S_{\hat{x}}\cdot S_{\hat{x}}}, & b_{(4)}=\dfrac{S_{\hat{x}\hat{x}}\cdot S_y-S_{\hat{x}}\cdot S_{\hat{x}y}}{n\cdot S_{\hat{x}\hat{x}}-S_{\hat{x}}\cdot S_{\hat{x}}} & \\
a_{(5)}=\dfrac{n\cdot S_{\frac{y}{x}}-S_{\frac{1}{x}}\cdot S_y}{n\cdot S_{\frac{1}{xx}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{x}}}, & b_{(5)}=\dfrac{S_{\frac{1}{xx}}\cdot S_y-S_{\frac{1}{x}}\cdot S_{\frac{y}{x}}}{n\cdot S_{\frac{1}{xx}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{x}}} & \\
a_{(6)}=\dfrac{n\cdot S_{\frac{x}{y}}-S_x\cdot S_{\frac{1}{y}}}{n\cdot S_{xx}-S_x\cdot S_x}, & b_{(6)}=\dfrac{S_{xx}\cdot S_{\frac{1}{y}}-S_x\cdot S_{\frac{x}{y}}}{n\cdot S_{xx}-S_x\cdot S_x} & \\
a_{(7)}=\dfrac{n\cdot S_{\frac{1}{xy}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{y}}}{n\cdot S_{\frac{1}{xx}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{x}}}, & b_{(7)}=\dfrac{S_{\frac{1}{xx}}\cdot S_{\frac{1}{y}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{xy}}}{n\cdot S_{\frac{1}{xx}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{x}}} & \\
a_{(8)}=\dfrac{n\cdot S_{\frac{\hat{y}}{x}}-S_{\frac{1}{x}}\cdot S_{\hat{y}}}{n\cdot S_{\frac{1}{xx}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{x}}}, & \hat{b}_{(8)}=\dfrac{S_{\frac{1}{xx}}\cdot S_{\hat{y}}-S_{\frac{1}{x}}\cdot S_{\frac{\hat{y}}{x}}}{n\cdot S_{\frac{1}{xx}}-S_{\frac{1}{x}}\cdot S_{\frac{1}{x}}}, & b_{(8)}=e^{\hat{b}_{(8)}} \\
a_{(9)}=\dfrac{n\cdot S_{\frac{x}{\hat{y}}}-S_x\cdot S_{\frac{1}{\hat{y}}}}{n\cdot S_{xx}-S_x\cdot S_x}, & b_{(9)}=\dfrac{S_{xx}\cdot S_{\frac{1}{\hat{y}}}-S_x\cdot S_{\frac{x}{\hat{y}}}}{n\cdot S_{xx}-S_x\cdot S_x} &
\end{array}\right]\quad (3)$$

To determine which of the nine models is more appropriate, we will associate the Pearson correlation coefficient with each model. (Baron *et al*, 1996, p.173), (Biji *et al*, 2000, pp.306-307) and so forth.

$$R_{(M)}=\sqrt{1-\frac{S_{up}}{S_{down}}}=\sqrt{1-\frac{\sum_{k=1}^{n}\left(y_k-y_{(x_k)}^{(M)}\right)^2}{\sum_{k=1}^{n}\left(y_k-\overline{y}\right)^2}}\in[0,1]\,,\quad M=\overline{1,9}\,,\ \text{where:}\quad \overline{y}=\frac{S_y}{n}=\frac{\sum_{k=1}^{n}y_k}{n}\quad (4)$$

The value of M indicates one of the nine mathematical models presented at the beginning of the paragraph in the relations (1).

For a model in which, under the radical, the sum from the numerator is greater than the sum from the denominator ($S_{up} > S_{down} > 0$), we will consider that particular model totally inadequate and we will give the value 0 to the correlation coefficient of the respective model.

The model with the highest correlation coefficient, closer to the value 1, is considered to be the optimal model.

4. Findings

In order to clarify how the mathematical relations in the previous paragraph are used, we will present a hypothetical case study and its solution.

In this respect, we generated nine groups of value pairs on the computer (nine clouds, each composed of 30 points): $P_k^{(a)}(x_k, y_k^{(a)})$, $P_k^{(b)}(x_k, y_k^{(b)})$, ..., $P_k^{(i)}(x_k, y_k^{(i)})$, $k = \overline{1,n}$, $n = 30$.

For each point cloud $P_k^{(L)}(x_k, y_k^{(L)})$, $L \in \{a,b,c,d,e,f,g,h,i\}$ the model that generates the optimal regression is desired to be determined.

For all nine groups, the values of the independent variable x are the same.

The computer-generated values are shown in tabular form below:

Table no. 1. Coordinates of the nine groups of points

k	x_k	$y_k^{(a)}$	$y_k^{(b)}$	$y_k^{(c)}$	$y_k^{(d)}$	$y_k^{(e)}$	$y_k^{(f)}$	$y_k^{(g)}$	$y_k^{(h)}$	$y_k^{(i)}$
–	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	10	79	13	4	119	313	1654	300	2183	33412
2	14	118	28	7	151	276	1073	367	772	1950
3	18	184	74	37	203	282	819	445	456	385
4	22	205	76	24	204	250	635	468	302	135
5	26	247	102	32	222	244	527	503	235	65
6	30	281	121	34	231	231	444	525	189	38
7	34	328	155	51	250	233	393	556	169	25
8	38	371	187	66	264	232	352	579	154	18
9	42	397	203	67	260	215	302	582	127	14
10	46	426	224	74	258	202	262	586	106	11
11	50	488	279	118	289	222	260	622	120	9
12	54	557	343	174	325	249	268	662	142	7.6
13	58	559	341	169	295	209	213	635	99	6.6
14	62	608	387	218	310	217	207	653	104	5.9
15	66	656	433	276	324	223	201	670	108	5.3
16	70	701	478	341	335	227	195	683	110	4.8
17	74	719	497	392	319	204	162	668	85	4.4
18	78	757	537	480	322	202	151	672	81	4.1
19	82	815	598	607	345	219	161	696	97	3.8
20	86	864	651	749	359	227	162	711	104	3.6
21	90	889	681	898	348	211	140	701	87	3.4
22	94	927	725	1095	351	209	132	703	83	3.2
23	98	945	750	1316	333	186	104	686	60	3.1
24	102	987	800	1616	339	188	101	692	61	3
25	106	1047	869	1999	363	207	116	716	80	2.8
26	110	1117	948	2472	397	237	142	749	109	2.7
27	114	1145	987	3002	388	224	126	741	96	2.6
28	118	1142	996	3621	349	181	79	701	52	2.6
29	122	1203	1070	4450	373	202	96	725	72	2.5
30	126	1275	1155	5469	408	233	125	759	103	2.4
$\sum$	2040	20037	14708	29858	9034	6755	9602	18756	6546	36136.4

Source: computer generated data

We aim to determine for each of the nine groups of values (point clouds) $(x_k, y_k^{(L)})$, $L \in \{a,b,c,d,e,f,g,h,i\}$ which model is more appropriate $y^{(L)} = f^{(L)}(x)$, where the function f is chosen from the nine mathematical models presented at the beginning of the previous paragraph.

During the first stage, the 123 sums of the values of the coordinates from the previous table, the logarithms thereof, of the inverses and of the products between them are calculated:

$$S_x = 2040,\ S_y^{(a)} = 20037,\ ...,\ S_y^{(i)} = 36136.4,\ S_{xx} = 174680,\ S_{xy^{(a)}} = 1723970,\ ...,$$

$$S_{xy^{(i)}} = 383269.6,\ S_{\hat{x}} = 121.13,\ S_{\hat{y}^{(a)}} = 189.2,\ ...,\ S_{\hat{y}^{(i)}} = 74.68,\ ...,\ S_{\hat{x}\hat{x}} = 502.56,\ ...,$$

$$S_{\frac{1}{x}} = 0.691,\ S_{\frac{1}{y^{(a)}}} = 0.0745,\ ...,\ S_{\frac{y^{(a)}}{x}} = 290.53,\ ...,\ S_{\frac{x}{y^{(a)}}} = 3.1065,\ ...,$$

$$S_{\frac{1}{\hat{y}^{(a)}}} = 4.827,\ ...,\ S_{\frac{x}{\hat{y}^{(a)}}} = 308.8\ ,\ ...,\ S_{\frac{1}{xy^{(a)}}} = 0.00327,\ ...,\ S_{\frac{\hat{y}^{(i)}}{x}} = 3.059\ .$$

All these amounts were calculated according to the corresponding relations (2).

During the next stage, the parameters are calculated according to relations (3) $a_{(M)}$ and $b_{(M)}$ which define the nine models.

For the first point cloud $P_k^{(a)}(x_k, y_k^{(a)})$ (L=a ,$k=\overline{1,30}$), the nine models are:

$a_{(1)} = 10.052$,	$b_{(1)} = -15.606$	$\rightarrow$	$\boxed{y_{(1)}^{(a)} = 10 \cdot x - 15.6}$	$R_{(1)}^{(a)} = 0.9991$
$a_{(2)} = 1.055$,	$b_{(2)} = 7.751$	$\rightarrow$	$y_{(2)}^{(a)} = 7.8 \cdot x^{1.1}$	$R_{(2)}^{(a)} = 0.9990$
$a_{(3)} = 0.019$,	$b_{(3)} = 147.58$	$\rightarrow$	$y_{(3)}^{(a)} = 148 \cdot e^{0.02 \cdot x}$	$R_{(3)}^{(a)} = 0.914$
$a_{(4)} = 493.33$,	$b_{(4)} = -1323.94$	$\rightarrow$	$y_{(4)}^{(a)} = 493 \cdot \ln x - 1324$	$R_{(4)}^{(a)} = 0.951$
$a_{(5)} = -13346.4$,	$b_{(5)} = 975.16$	$\rightarrow$	$y_{(5)}^{(a)} = 975 - \frac{13346}{x}$	$R_{(5)}^{(a)} = 0.791$
$a_{(6)} = -0.0000545$,	$b_{(6)} = -0.000273$	$\rightarrow$	$y_{(6)}^{(a)} = -\frac{1}{0.00005 \cdot x + 0.0003} = -\frac{20000}{x+6}$	$R_{(6)}^{(a)} = 0$
$a_{(7)} = 0.122$,	$b_{(7)} = -0.000316$	$\rightarrow$	$y_{(7)}^{(a)} = \frac{x}{0.122 - 0.0003 \cdot x}$	$R_{(7)}^{(a)} = 0.949$
$a_{(8)} = -32.475$,	$b_{(8)} = 1157.83$	$\rightarrow$	$y_{(8)}^{(a)} = 1158 \cdot e^{-\frac{32.5}{x}}$	$R_{(8)}^{(a)} = 0.901$
$a_{(9)} = -0.000539$,	$b_{(9)} = 0.1976$	$\rightarrow$	$y_{(9)}^{(a)} = e^{\frac{1}{0.2 - 0.0005 \cdot x}} = e^{\frac{2000}{400 - x}}$	$R_{(9)}^{(a)} = 0.560$

Since $\max\limits_{M=\overline{1,9}} R_{(M)}^{(a)} = R_{(1)}^{(a)} = 0.9991$, for the first cloud the linear model that is marked above is appropriate.

For the second and third clouds (L=b and L=c), the most appropriate are the power model and the exponential model.

$P_k^{(b)}(x_k, y_k^{(b)})$, L=b		$P_k^{(c)}(x_k, y_k^{(c)})$, L=c	
$y_{(1)}^{(b)} = 9.7 \cdot x - 171$	$R_{(1)}^{(b)} = 0.992$	$y_{(1)}^{(c)} = 33.6 \cdot x - 1288$	$R_{(1)}^{(c)} = 0.821$
$\boxed{y_{(2)}^{(b)} = 0.46 \cdot x^{1.62}}$	$R_{(2)}^{(b)} = 0.997$	$y_{(2)}^{(c)} = 0.004 \cdot x^{2.8}$	$R_{(2)}^{(c)} = 0.825$
$y_{(3)}^{(b)} = 44 \cdot e^{0.03 \cdot x}$	$R_{(3)}^{(b)} = 0.817$	$\boxed{y_{(3)}^{(c)} = 6.65 \cdot e^{0.054 \cdot x}}$	$R_{(3)}^{(c)} = 0.988$
$y_{(4)}^{(b)} = 460 \cdot \ln x - 1366$	$R_{(4)}^{(b)} = 0.908$	$y_{(4)}^{(c)} = 1387 \cdot \ln x - 4605$	$R_{(4)}^{(c)} = 0.657$
$y_{(5)}^{(b)} = 764 - \frac{11886}{x}$	$R_{(5)}^{(b)} = 0.723$	$y_{(5)}^{(c)} = 1709 - \frac{31012}{x}$	$R_{(5)}^{(c)} = 0.452$
$y_{(6)}^{(b)} = -\frac{100000}{83 \cdot x + 27}$	$R_{(6)}^{(b)} = 0$	$y_{(6)}^{(c)} = -\frac{100000}{24 \cdot x + 27}$	$R_{(6)}^{(c)} = 0$
$y_{(7)}^{(b)} = \frac{1000 \cdot x}{653 - 8 \cdot x}$	$R_{(7)}^{(b)} = 0$	$y_{(7)}^{(c)} = \frac{100 \cdot x}{224 - 3 \cdot x}$	$R_{(7)}^{(c)} = 0$
$y_{(8)}^{(b)} = 1054 \cdot e^{-\frac{51}{x}}$	$R_{(8)}^{(b)} = 0.874$	$y_{(8)}^{(c)} = 1639 \cdot e^{-\frac{78.6}{x}}$	$R_{(8)}^{(c)} = 0.359$
$y_{(9)}^{(b)} = e^{\frac{1}{0.26 - 0.0012 \cdot x}} = e^{\frac{2500}{650 - 3 \cdot x}}$	$R_{(9)}^{(b)} = 0$	$y_{(9)}^{(c)} = e^{\frac{10000}{4000 - 27 \cdot x}}$	$R_{(9)}^{(c)} = 0$

The appropriate models for the two cases (L=b and L=c), were chosen because $\max_{M=1,9} R_{(M)}^{(b)} = R_{(2)}^{(b)} = 0.997$ and $\max_{M=1,9} R_{(M)}^{(c)} = R_{(3)}^{(c)} = 0.988$.

For the other six groups of value pairs (the cases $L \in \{d,e,f,g,h,i\}$) the optimal regressions are obtained by similar calculations and they are, in order, a logarithmic model, three hyperbolic models of type 1, 2 and 3 and two exponentially-hyperbolic models of type 1 and 2.

At the end of the paragraph we should mention that the nine groups with value pairs (point clouds) had as appropriate regressions nine models of different shapes:

$y^{(a)} = 10 \cdot x - 15.6$ linear model $y^{(b)} = 0.46 \cdot x^{1.62}$ power model

$y^{(c)} = 6.65 \cdot e^{0.054 \cdot x}$ exponential model $y^{(d)} = 103 \cdot \ln x - 113$ logarithmic model

$y^{(e)} = 198 + \frac{1168}{x}$ hyperbolic model (type 1)

$y^{(f)} = -\frac{100000}{8 \cdot x - 27}$ hyperbolic model (type 2)

$y^{(g)} = \frac{1000 \cdot x}{1.2 \cdot x + 21}$ hyperbolic model (type 3).

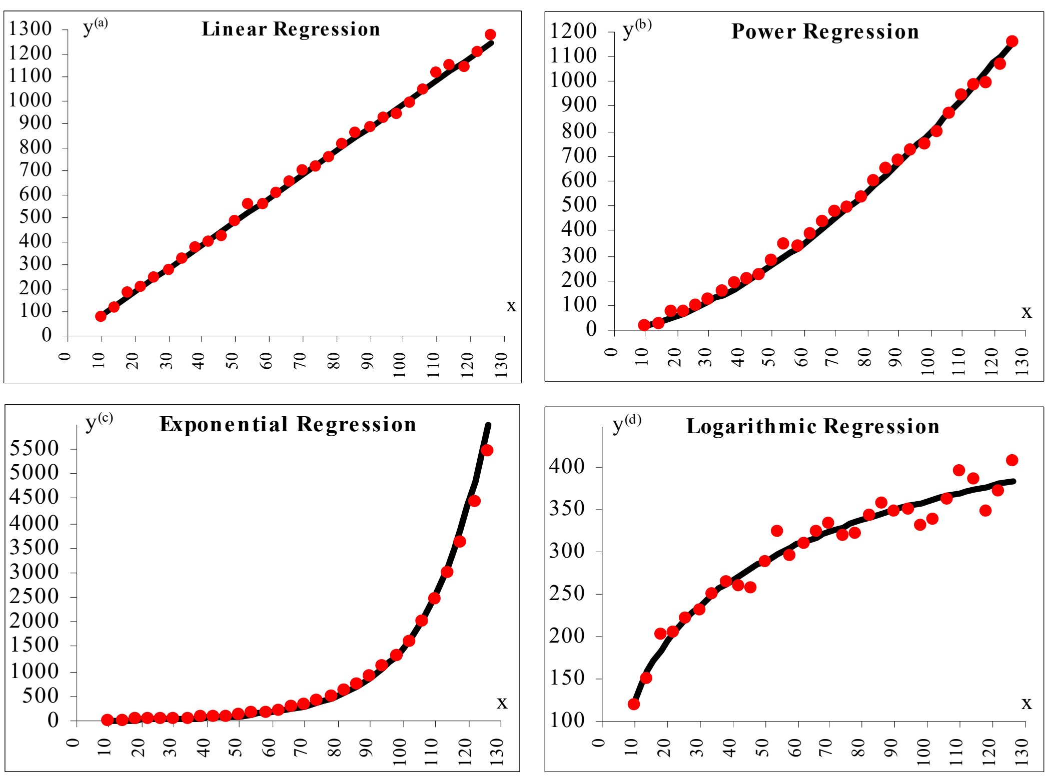
$y^{(h)} = 57 \cdot e^{\frac{36.6}{x}}$ hyperbolic exponential model (type 1)

$y^{(i)} = e^{\frac{10000}{90 \cdot x + 69}}$ hyperbolic exponential model (type 2).

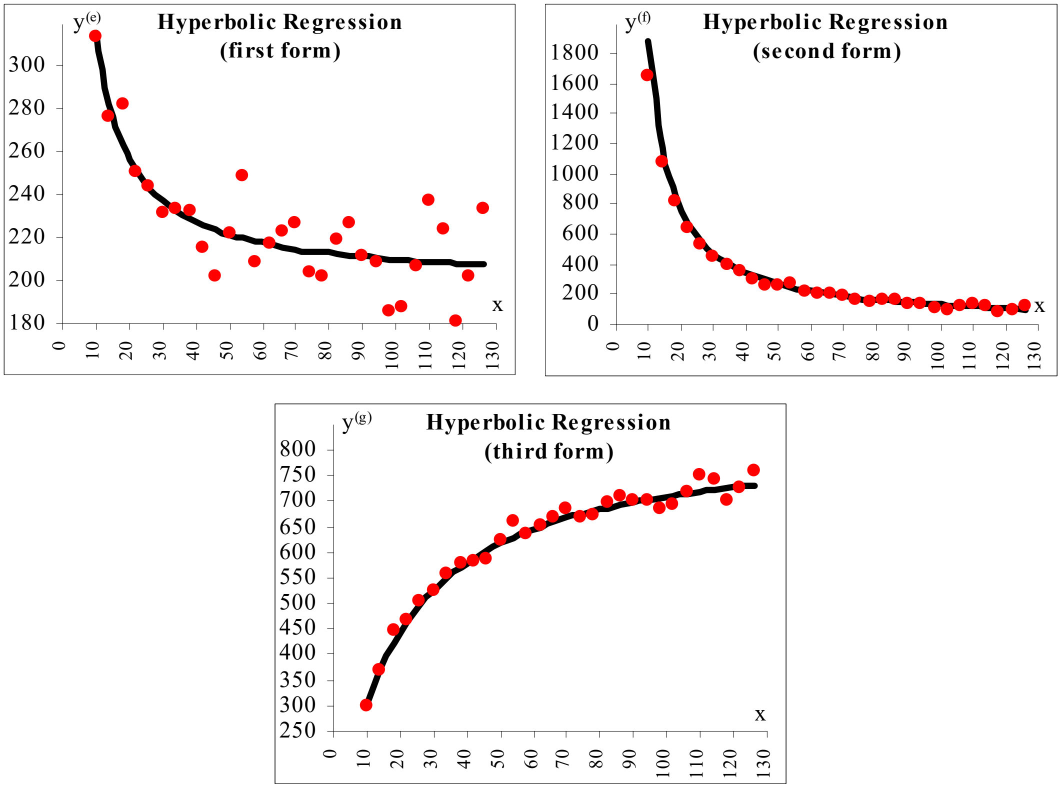
5. Conclusions

In order to establish the validity and credibility of the method for obtaining the nine simple regressions, we represented each point cloud and the calculated optimal regression in nine different graphs.

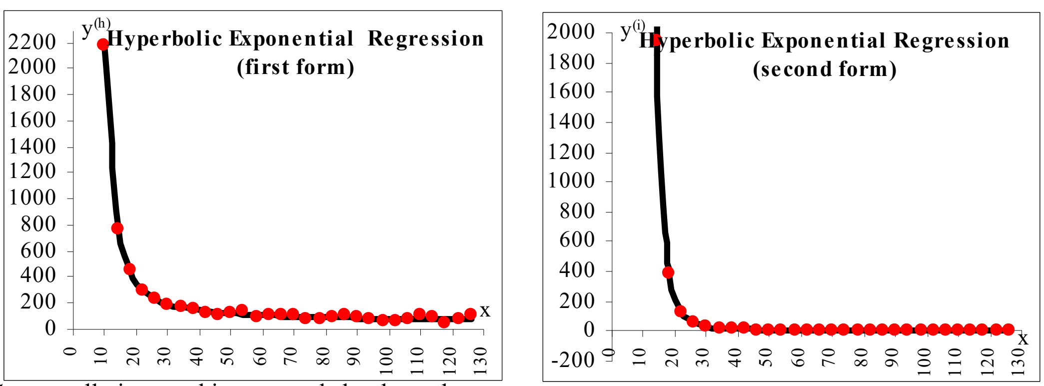
Graph no. 1. The graphical representation of regressions: linear, power, exponential and logarithmic regressions



Graph no. 2. The graphical representation of the three forms of hyperbolic regressions



Graph no. 3. The graphical representation of exponential-hyperbolic regressions



Source: all nine graphics are made by the author

The previous graphs clearly show that the method used is valid. We believe that the technique applied throughout the paper can become a valuable tool available to all researchers both in the economic and social field and in other scientific fields.

In this regard, we hope that in the near future we will publish a paper with the logical scheme and the pseudo-code for the method applied in this paper, which are elements that will be the basis for the achievement of an application by the specialists in the field of IT.

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