

Assessing Portfolio Risks Involving Bitcoin and Ethereum Using Vector Autoregressive Model

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Abstract

Investors now have a multitude of non-traditional assets to choose from, especially from the spectrum of alternative assets, such as financial digital assets.

We start from the premise that there is a high risk associated with investing in financial digital assets, along with the opportunities presented from these emerging digital markets that evolve in a decentralized environment. We will be looking at the two major digital assets, specifically Bitcoin (BTC) and Ethereum (ETH), as per their dominance within the markets of crypto assets.

This paper will focus on the evolution of financial digital assets and the impact on portfolio assessment that have allocations for BTC and ETH. In order to identify the value and potential of these financial digital assets, we will be addressing volatility and portfolio risks by means of a Vector Autoregression model on the returns of both, BTC and ETH.

Key words: financial digital assets, volatility, portfolio risk

J.E.L. classification: F33, F42, F01, E02

1. Introduction

Technological development has produced enormous opportunities for investors to invest in non-traditional assets such as alternative assets from the crypto space. The major drawback of traditional assets is the extent of responsiveness to changing macroeconomic and global financial conditions. However, Bitcoin, one of the many cryptocurrencies from the financial digital assets space, appeared to be a haven against changing macroeconomic and global financial conditions (Bouri et al., 2018). In the same way, Bitcoin is out of the purview of a centralized institutions such as governments, banks, or other financial intermediaries. The decentralized nature of Bitcoin incentivizes the environment for investors, and there is a spectacular rise in interest towards these type of financial digital assets. For instance, the mean annual return of Bitcoin from 2013 to up to date stood at 408.8% (UpMyInterest, 2021), which is much higher than the annual return of many other assets in the crypto space and outside of this ecosystem.

The concept behind the successful growth and adoption of Bitcoin, as it roughly captures almost 41.5% of total market capitalization in the crypto market (Coinmarketcap, 2021). Bitcoin appeared to be the first decentralized and unregulated financial digital asset, which attracted massive attention from media, academics and the financial industry along with its investors. In November 2021, Ethereum price reached the \$4800 level, and it is second most traded financial digital asset in the market with a market share dominance of 23% (Coinmarketcap, 2021).

This research study focuses on assessing the degree of price volatility and portfolio risk associated with Bitcoin and Ethereum. We will be conducting an empirical study to measure the price volatility of these two different assets in the crypto assets' environment.

2. Literature review

There exist different strands of literature related to financial digital assets. The first strand of literature deals with the contribution of crypto assets to portfolio diversification. For instance, Ozturk (2020) suggested that Bitcoin might not provide sufficient contributions to portfolio diversification

in the short and medium-term, mainly due to the volatile nature of Bitcoin. However, due to limited connectedness between Bitcoin and other assets, like gold and crude oil, it might offer potential gains from diversification in the long run. The study implies that crypto assets are significant for portfolio diversification, especially on a longer timeframe. Similarly, Conlon et al. (2020) modeled and examined most international equity markets, and concluded that crypto assets do not appear to be a safe haven except for the Chinese CSI 300 index. For Zhang & Ding (2021), frequent and high fluctuations in cryptocurrency prices, induce investors to thoroughly account for risk portfolio assessments. However, the volatility impact on the market varies from short, medium to long runs with different performance scenarios. The negative impact on the market is more severe in both, the short and medium timeframe, but the risk of spillover gets down in the long run.

The second strand of literature deals with crypto as an alternative to FIAT currencies. After being accepted as a medium of exchange, Bitcoin failed to gain momentum in retail transactions for significant reasons. First, Bouri et al. (2019) argued that Bitcoin is not regulated. Second, since no central authority is navigating the price movement, high volatility is expected in the price and returns of Bitcoin. Alternatively, decentralization and lack of regulation allow investors to face frequent steep ups and downs in this type of asset's price shocks and returns. Due to this reason, it is less plausible that crypto assets would be used as a currency substitute since they lack exposure to currency and commodity returns. For instance, Liu & Tsyvinski (2021) findings posed severe challenges to the popular explanations that crypto in its existing shape cannot serve as a unit of account or store of value due to its lack of exposure to the returns of earnings on foreign exchange and commodity investments. For Brenyah (2018), the lack of acceptance by institutional investors to engage in the ecosystem makes the crypto market more volatile and serves as an alternative to currencies.

The third strand of literature is related to the price and return volatility of crypto assets. This segment of literature explains and focuses on the risk associated with cryptocurrencies in different scenarios and studies. For instance, annualized return volatility of Bitcoin has stood at 81% since 2013, and this is likely to be due to the lack of interest from the institutional investors who still consider Bitcoin a speculative asset. In the absence of normal assets return distribution, variance is no longer a good measure of portfolio risk. The daily cryptocurrency returns do not exhibit a normally distributed function but follow the Cauchy distribution function (Hrytsiuk et al., 2019).

Different econometric models and techniques have been used to forecast crypto volatility to help crypto investors' decision-making and risk management. For Instance, Brenyah (2018) used Value at Risk (VaR), expected shortfall, and filtered historical simulation with the help of GARCH modeling. According to Kyriazis (2021), sophisticated GARCH models are much more efficient in explaining the fluctuations in the volatility of digital financial assets. However, the author has shown that the inclusion of Bitcoin with conventional assets significantly changes the risk-return trade-off involved in the decisions of different investors. Similarly, Wang (2021) findings revealed the haven property of Bitcoin and argued that Bitcoin could provide a strong hedge against recessionary periods. Specifically, during economic recessions, portfolio risk will decrease if Bitcoin, as an asset, is part of the investor's portfolio. Shen et al. (2021) applied both conventional modeling and machine learning to forecast volatility in the returns of bitcoin, and Recurrent Neural Network (RNN) outperforms Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) and Exponentially Weighted Moving Average (EWMA).

Apart from GARCH and its variant models, Value-at-Risk (VaR) measure has been frequently used to assess the volatility of Bitcoin's return and its contribution to the portfolio risk. For instance, Stavroyiannis (2018) concluded that because of high volatility in Bitcoin's returns, the distribution is not normal, and for that reason, the VaR approach is not suitable. However, Conditionally Adjusted VaR (CAVaR) models effectively explain the risk connected to different crypto assets. In this regard, Li & Huang (2020) examined risk connectedness among seven major cryptocurrencies using the CAVaR model and shown that risk spillover levels are stronger under downward risk than upward risk tendency.

3. Research methodology

Our approach for this empirical study will involve a quantitative research methodology based on the Vector Autoregression (VAR) model to achieve our objectives and to assess the level of portfolio risk associated with digital financial assets.

Our study will identify the optimal lag for our two sets of time series, daily Log returns of Bitcoin and Ethereum. The VAR model enables the study to obtain coefficient matrices, the covariance matrix of residuals, and impulse response function. First, we will calculate the Log daily return of Bitcoin and Ethereum by using daily closing and opening prices. And then, in the second stage, we will estimate the VAR model and detail the obtained results through diagnostic tests and impulse response graphs.

The research strategy is structured for the study to provide coefficient matrices and explain each coefficient's statistical significance. Results and comments on the covariance matrix of residuals will be provided, and finally, we will be presenting a conclusion after explaining the impulse response function. Vector autoregressive model extends univariate regression and involves more than one-time series, whereas univariate autoregression is suited for single time series. In the VAR model, a vector of variables is included in the model, and each variable depends on its lags and the lags of another variable in the vector.

4. Data Description

The timestamps for the data involved in this study have been collected from January 2016 to October 2021 and its based on the daily closing and opening prices of Bitcoin and Ethereum assets. For replication purposes, the data is public and can be extracted from many digital assets providers. Regarding the validity of data, we have engaged the data sets from Yahoo Finance, as the values are different for each provider due to the phenomena that most of the biggest crypto exchanges have and run their order books. Because of this, differences in price ranges are expected, and results may vary.

5. Econometric Model

Based on the knowledge we obtained reviewing the existing literature in this field, the available econometric models used in this type of analysis are the Value-at-Risk (VaR) approach, GARCH, and Vector Autoregressive model (VAR). However, we are using the VAR model assuming that both sets of time series, i.e., daily annualized Log returns of Bitcoin and Ethereum, are stationary at level, which entails properties of both time-series do not dependent on time and mean, and variance are consistent over time. Vector autoregressive model extends univariate regression, known as the single equation model where current values are determined by previous year value called lagged value. A vector of variables is included, and each variable depends on its lags and the lags of another variable in the vector. Equations 1 and 2 present the VAR model used for Bitcoin and Ethereum in this study:

$$B_t = \alpha_0 + \alpha_1 B_{t-1} + \alpha_2 E_{t-1} + \mu_1 t \quad (1)$$

$$E_t = \beta_0 + \beta_1 B_{t-1} + \beta_2 E_{t-1} + \mu_1 t \quad (2)$$

The two equations show a two-variable VAR with a single lag for both, Bitcoin and Ethereum. In the first equation, Bitcoin as a dependent variable depends on intercept, equal to the average daily log-returns of Bitcoin when there is no independent variable included in the model.

It is possible to expand the model to include more than one lag, which implies that latest price of bitcoin will be affected by the price of bitcoin from the previous day, and the same holds for Ethereum and the residuals, which should not be connected with the delays included in this particular model. The second equation depicts Ethereum as a dependent variable based on intercept (constant), lag (prior days prices), and residuals.

6. Results and Discussion

In VAR model analysis, checking the stationarity of the series is essential. According to Elder and Kennedy (2001), incorrect conclusions might be obtained if data stationarity is not validated. In the Dickey-Fuller test, autocorrelation in the error term prevents a first-order autoregressive process from being represented. The "Augmented Dickey-Fuller" test is used to check unit root in the model and its stationarity. (Göktaş, 2005). The series stationarity is assessed using ADF test statistics and within the stationary test for Bitcoin and Ethereum, the ADF Z- test value is less than the MacKinnon critical values for 1%, 5%, and 10% significance levels.

To eliminate the unit root, the first-degree difference in the series was collected and the stationary test was performed many times. If the H0 hypothesis is rejected, the series is seen as being stationary in nature.

Within table 1, the values of ADF test of stationarity are presented.

Table no. 1 Augmented Dickey Fuller test for Bitcoin and Ethereum

ADF FOR	Test statistics	1% critical value	5% critical value	10% critical value	MacKinnon approximate p-value for	Stationarity level	
ETH	Z(T)	-32.696	-3.43	-2.86	-2.57	0	Stationary (1)
BTC	Z(T)	-33.008	-3.43	-2.86	-2.57	0	Stationary (1)

Source: Author's creation

These results confirming that after taking 1st difference of Bitcoin and Ethereum are stationary at 1st difference as their p-value is less than 0.05.

To use the VAR model on the data, we need to determine the ideal lags, which indicates that today's price will be influenced by the price of the previous day. Tables 2 and 3 present obtained results of selection order criterion to select optimal lags for both BTC and ETH annualized return of log prices. Different types of tests have been applied to choose an optimal lag selection, for instance, log-likelihood (LL), likelihood Ratio Test (LR), Final Prediction Error (FPE), Akaike's Information Criterion (AKI), Hannan and Quinn Information Criterion (HQIC), and Schwarz's Bayesian Criterion (SBIC). Both FPE and AIC are used to estimate prediction error for various models and see the relative quality of each model.

If the lag duration is too short, the model's specification will be incorrect, and if it is too long, the degree of freedom will be of no benefit. The criteria we are going to use for lag length is HQ, Schwarts and Akaike.

All the mentioned tests are used to identify and choose optimal lag, and the basic rule of thumb to select optimal lag is to see the highest value for each mentioned type of test. For instance, as per the log-likelihood ratio test, the highest value is recorded at lag 4, which is accurate for all other types of tests.

One can see that the AIC criterion also shows lag 4 is optimal because, the value -9.82788 is the largest among the rest of the lags. Thus, the optimal lag for log opening and closing price would be lag 4.

Table no. 2 Optimal lag selection for Ethereum and bitcoin

Selection-order criteria					Number of observations = 2137			
lags	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	9417.66				5.10E-07	-8.81203	-8.81009	-8.80673
1	9417.66	1242.1	4	0	2.90E-07	-9.3895	-9.38368	-9.37359
2	10038.7	524.21	4	0	2.30E-07	-9.63106	-9.62136	-9.60454
3	10300.8	265.31	4	0	2.00E-07	-9.75147	-9.73788	-9.71434
4	10433.4	171.3*	4	0	1.8e-07*	9.82788*	9.81041*	-9.78015*

Endogenous: dlreturneth dlreturnbtc
Exogenous: constant

Source: Author's creation

In table 2, the optimal lag selection for difference of Ethereum' and Bitcoin annualized log of return prices is presented.

Using the logic mentioned earlier, it can be said that the optimal lag for Ethereum' and Bitcoin is set to be lag 4 represented by the asterisk sign. The term constant in the table represents our single exogenous variable in the model. Its value is determined outside of the model and shows the average value of the dependent variable when the model contains no independent variable. At the same time, the endogenous variable is determined within the model and, in this case, difference of annualized return of log prices for Bitcoin and Ethereum. Finally, the values holding stars also show that lag 4 is the optimal lag for this model using HQIC, AIC and SBIC criteria.

Table no. 3 Coefficient matrices

Equation	Parms		RMSE	R-sq	chi2	P>chi2
dlreturneth	9		.02796	0.3870	1349.18	0.0000
dlreturnbtc	9		.019016	0.4134	1506.167	0.0000
	Coef.	Std.Err.	z	P>z	[95%Conf.	Interval]
dlreturneth						
dlreturneth						
L1.	-0.769	0.026	-29.140	0.000	-0.820	-0.717
L2.	-0.578	0.032	-17.970	0.000	-0.642	-0.515
L3.	-0.354	0.032	-11.020	0.000	-0.417	-0.291
L4.	-0.175	0.026	-6.620	0.000	-0.226	-0.123
dlreturnbtc						
L1.	-0.018	0.039	-0.470	0.636	-0.094	0.058
L2.	-0.022	0.048	-0.460	0.644	-0.117	0.072
L3.	-0.079	0.048	-1.630	0.103	-0.173	0.016
L4.	-0.061	0.039	-1.560	0.118	-0.137	0.015
constant	0.000	0.001	0.040	0.968	-0.001	0.001
dlreturnbtc						
dlreturneth						
L1.	0.020	0.018	1.140	0.254	-0.015	0.056
L2.	0.008	0.022	0.350	0.723	-0.035	0.051
L3.	0.031	0.022	1.430	0.154	-0.012	0.074
L4.	0.014	0.018	0.810	0.419	-0.021	0.050

<hr/>						
dlreturnbtc						
L1.	-0.836	0.026	-31.740	0.000	-0.888	-0.785
L2.	-0.624	0.033	-19.040	0.000	-0.688	-0.560
L3.	-0.432	0.033	-13.180	0.000	-0.496	-0.368
L4.	-0.219	0.026	-8.310	0.000	-0.271	-0.167
constant	0.000	0.000	0.000	0.999	-0.001	0.001

Source: Author's creation

After choosing optimal lag for equations 1 and 2, in table 3, we present coefficient matrices obtained using the VAR model. Based on equation 1, assuming ETH Log of return on price as the dependent variable, it can be said that lag 4 of log return on price is significantly related to ETH log return on price. For instance, one can see that the coefficient is negative -0.769, and its associated probability value is less than the standard 5%.

It might be taken as: the past four-day return on ETH prices can predict latest price calculated in our study. A rise of 1% in the four-days lagged return on log of price reduces current prices by 0.769%, although these are negligible changes. For this reason, there is no statistical relevance to the remainder of the ETH price log, which is why it has no predictive value. In other words, the past four days BTC price movements may be used to anticipate latest price from our study. Our latest prices will drop 0.018% if a 1% rise in four-days lagged return on log of price occurs. Using the previous four days of price returns, these figures may be used to anticipate our latest price return, from our research.

There are two constant terms in the table, one in the log of return price of ETH equation and the second in the log return on price of BTC equation.

In both cases, they represent the average value of the dependent variable without considering for independent variable. For instance, in the both equations, the constant is equal to 0.000, which shows the average log of return on price of BTC and ETH when there is no independent variable.

Table no. 4 VAR Stable: Eigenvalue stability condition

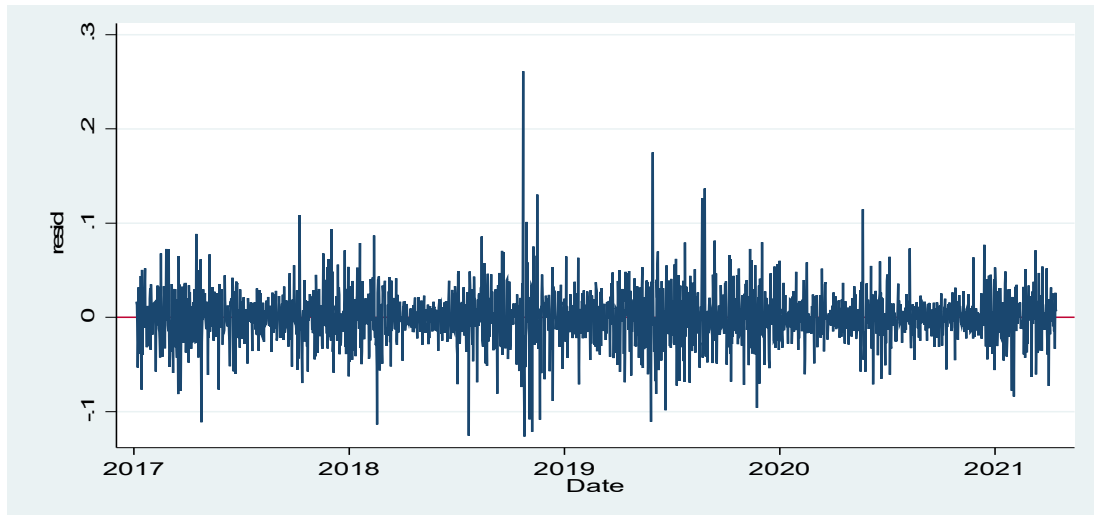
Eigenvalue	Modulus
.1144018 + .6917044i	.701101
.1144018 - .6917044i	.701101
.1559007+ .6568266i	.675075
.1559007- .6568266i	.675075
.5421565+ .382316i	.6634
.5421565 - .382316i	.6634
.5306347+ .3387802i	.62956
.5306347 - .3387802i	.62956

All the eigenvalues lie inside the unit circle. VAR satisfies stability condition.

Source: Author's creation

For the VAR model, table 4 shows that the outcomes are stable, which demonstrates that the model is stable. It is impossible to conduct an accurate test or estimate the standard error of the impulse response function if the VAR is not steady. This test shows that all of the values are inside the unit circle and that VAR stabilizes the model, which is important for our study.

Figure no. 1 Residual line graph around the mean



Source: Author's creation

This graph from figure 1 confirms that the residual is showing appropriate estimated VAR model and confirms the stationarity and stability of the model. The graph depends on residual and years of the data from 2017-2021 all daily observations are around the mean value except some of them which are outliers.

Table no. 5 Granger Causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2	Results
dlreturneth	dlreturnbtc	3.9837	4	0.408	Btc does not affect price of eth
dlreturneth	ALL	3.9837	4	0.408	There is no causal relation
dlreturnbtc	dlreturneth	3.9965	4	0.406	Eth does not affect price of btc
dlreturnbtc	ALL	3.9965	4	0.406	There is no causal relation

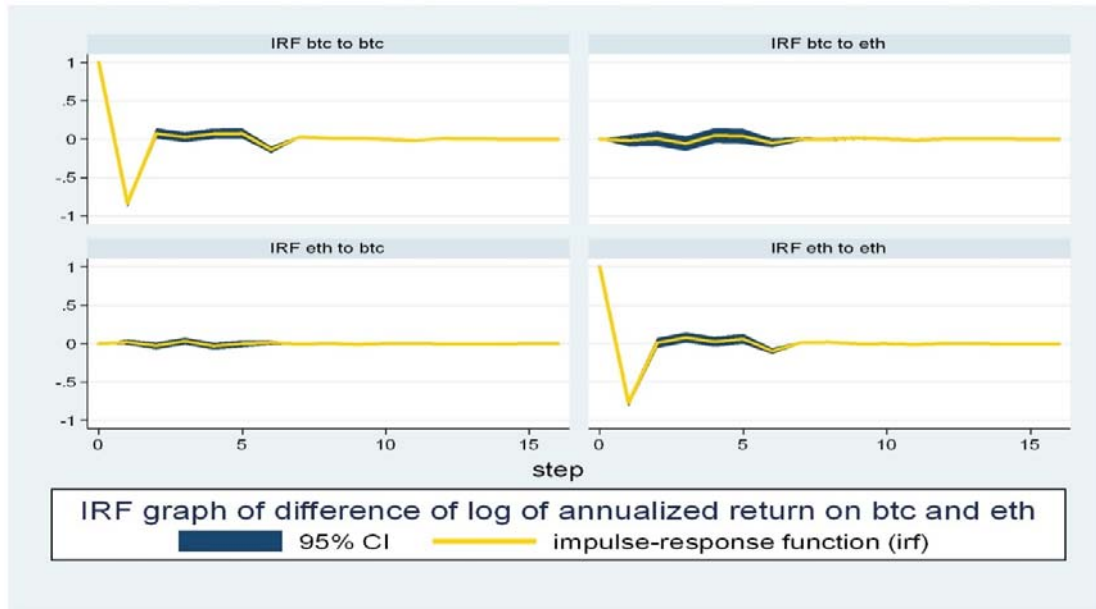
Source: Author's creation

A causality test must be done to assess the direction of the association between two variables. Causality is the statistical notion that future predicted values of a time series variable are influenced by previous period values.

This element of causality explores if the delayed values from a variable may be utilized as an explanation for a different variable. Granger test is thought to be induced by variables (x) if the delayed values of (x) have a significant impact on variables (x, y).

In this model, since all the probability values are more than 0.05, no other variable impact, such as BTC or ETH, was able to forecast the return of ETH prices or the return of BTC prices. Neither of the variables are connected in any way.

Figure no. 2 Impulse Response Function return on prices of BTC to ETH



Source: Author's creation

The impulse response function is used to show the impact of the exogenous shock on the system equation. There are two different variables used in impulse response function: impulse and response variable. For instance, in figure 2, the return on price for BTC is treated as an impulse variable and the return of price for ETH as response variable. The figure shows the effect of a one-standard-deviation impulse on the return of log price of the BTC equation. After the initial impulse, the log of return on price shows extreme fluctuations and IRF from BTC to ETH is wider than all other impulses.

The second row in figure 2 shows the effect impulse from ETH to BTC is minimum because the line is straight almost with no variations and reaches to zero and it does not have any wider effect. In the last graph impulse response of ETH to ETH start from one which means start from previous lag value and then it declines to minimum value after that it becomes a straight line on zero.

7. Conclusions

For this research we have selected the two major dominating financial digital assets of the crypto market. Based on our study we can conclude that the price movements, for both Bitcoin and Ethereum, within the spectrum of alternative financial assets, are momentum driven and involve a high volatility. As we are approaching a portfolio risk assessment, in our opinion the risk is high and is directly correlated with the returns of these assets. Thus, the allocations for this kind of assets in such a scenario should be carefully calculated, as every portfolio manager will be looking the best returns on their investments.

Clearly, the two assets studied above, make a great contribution to a portfolio and the returns are substantial and present a great investment opportunity.

Our results confirm that the prices for both Ethereum and Bitcoin are independent, but we can ascertain correlations. In our opinion the implications of price fluctuations and the elasticity for shock absorption, for each of our assets, influence one another. We believe this is a direct implication on how the crypto market evolved, in terms of accumulating liquidity along with the way the trading pairs have developed over time. Based on our research on price fluctuations and volumes, presented on different crypto exchanges, we can clearly notice that most the leading pairs for trading, start with ETH/BTC as leading pair and the rest of the crypto assets just follow the trend, trading against BTC and ETH. As the market started to mature and more players and projects have been developed, the liquidity and volumes started helping the independency of these two assets. A major pivot for the

volatility and liquidity, for both BTC and ETH, was the introduction and adoption of the stablecoins, which allowed these assets to define their trading value in USD directly.

There is an important aspect that we need to mention with regards to the data drawn for our study and that is the timeframe of the data used. We have conducted our research based on data from 2016 to 2021, for the reason that the influx of liquidity for these assets has started at higher rates from late 2020. It's an important aspect as the prices, liquidity and volatility related to these assets from 2016 till 2020 was quite constant with no major spikes or abrupt price fluctuations.

The reason for this set of data is to assess the volatility over these two major liquidity periods, that directly influence the volatility assessment. Our returns analysis and results present an independency between these assets, but with a liquidity evolution and correlation.

Investors risk assessment on portfolio allocations that involve financial digital assets from the crypto sphere should be carefully vetted as they are to be treated as high-risk investments with great returns.

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