Uncertainty Management Using Triangular Fuzzy Numbers with Associated Variable Indicators

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Abstract

Any decision-making process encounters many difficulties in ensuring the necessary amount of information (the available knowledge being incomplete and insecure, usually in different proportions), respectively the way of determining the optimal decision-making variant, achieving consensus on how to implement.

Therefore, the process of making decision under conditions of uncertainty required the development of new ways of its reduction or absorption.

It is important to notice that from a practical point of view, the fuzzy numbers are closer subjects for the logic argumentation of an action course specific for IIIrd degree uncertainty conditions, that will ultimately support new perspectives for an easier manner of information modelling in regard with economic and social phenomena.

In therefore makes sense the increasing concern for the above mentioned specific instruments theoretical framework developments. The current paperwork considers the uncertainty analysis within the management processes, by using the triangular fuzzy numbers with variable associated indicators, by actually using an uncertainty specific associated absorption indicator, along with elementary operations. Furthermore, an optimal decision analysis, specific for random values of the absorption of the uncertainty indicator it is considered, within different methods.

Key words: decision making, uncertainty, triangular fuzzy numbers, index of uncertainty management
J.E.L. classification: D81, M10

1. Introduction

Uncertainty is the factor that exposes any economic or social system to many situations of crisis or opportunity, the difference between the two antagonistic situations being made only by senior management through visionary quality, experience and the volume of material and human resources available to the company at a time. dat.

It is thus necessary to intensify the theoretical efforts that through interdisciplinary approaches and methodological loans to structure a complex of procedures and techniques that will contribute to the diminution of this phenomenon within the economic decision-making processes.

To reduce uncertainty, several strategic management directions have been crystallized so that entrepreneurs can show a logically structured attitude in extreme situations. In practice, an important place is occupied by the technocratic methods which consist in the use of the best forecasting techniques, to anticipate future contexts and to elaborate specific contingency plans.

In the case of our paper, the focus was to develop a new model for making decisions in conditions of persistent uncertainty (degree III) starting from the formal or informal methods of managing uncertainty, by using triangular fuzzy numbers with associated variable indicators.

The main steps in the use of fuzzy numbers in the decision-making process are proposed:

1. Brief structure of the context / activity based on input data, socio-economic and ethical information
2. defining the problem by analysing the organizational objectives, identifying the difficulties or gaps

3. formulation of alternative solutions
4. assessing the consequences and formalizing alternative solutions by fuzzy numbers which involves:
   – determining the values to be kept or promoted
   – identification of the rules, laws and regulations that apply
   – evaluation of the consequences through quantitative and qualitative characteristics of the alternative solutions, of their uncertainty and subjectivism
   – choosing the best method for formalizing information by fuzzy numbers (by interval, triangular number, trapezoidal number, etc.)
   – formalization of information through a matrix of variants and consequences
5. hierarchy of alternative solutions
   – o identifying the best method for ranking fuzzy solutions (simple ordering, distance from an optimal, etc.)
   – o hierarchy of solutions.
6. choosing the optimal solution based on an optimal criterion.
7. implementation of the solution
8. evaluation of the approaches for solving the problem (feedback)

Even though the main features that underpin the evaluation and description specific for a situational decision vary across domains (economic, social, political etc.). Finally, it can be retained a definition that relies on the approach specific for the decisional context development, while the information quantity and quality leads towards identifying decisional consequences that are difficult to anticipate and control, while the specific uncertainty is hard to absorb.

Thus, it is proposed to develop a new theoretical direction to approach the decision-making processes in conditions of uncertainty and specific tools much closer to the practical way depending on an index for management of uncertainty.

2. Theoretical background

We can talk about many approaches regarding a representation of a **triangular fuzzy number** (**NFT$_r$**). Starting from the definition given by Zadeh (1975) and up to the many theoretical and practical developments utilized in the last decades are useful to our demonstration. In view of the objective pursued, a **NFT$_r$** may be represented simply by an ordered triplet of the form $<T>=(T_s,T_m,T_d)\in$NFT$_r$, where $T_s,T_m,T_d\in \boldsymbol{R}$, defined having the membership function $\mu_T:\boldsymbol{R}\rightarrow[0, 1]$, as follows (Bodjadziev, Maturo, Tofan]:

$$\mu_T=\begin{cases}\dfrac{x-T_s}{T_m-T_s}, & T_s\le x\le T_m\\ 1, & x=T_s\\ \dfrac{T_d-x}{T_d-T_m}, & T_s\le x\le T\\ 0, & x\notin[T_s,T_d]\end{cases} \quad (1)$$

We will present some of the most representative and suggestive ways to perform operations with triangular fuzzy numbers.

Elementary operations with triangular fuzzy numbers:

Let there be triangular fuzzy numbers with variable centers of gravity:

$<T1>=(T1_s,T1_m,T1_d)$, $<T2>=(T2_s,T2_m,T2_d)$ and $<T3>=(T3_s,T3_m,T3_d)\in$ NFT$_r$

Starting from the exhaustive way developed (Zadech, 1975; Pride), with various contributions from many contributors (Bodjadziev, 1995, p 85; Jacobsen, 2004, p82) the following addition, multiplication, subtraction and division operations have been described in a more accessible form for practical applications:

The addition

$$< T3 >=< T1 > \oplus < T2 >= (T1_s + T2_s; T1_m + T2_m; T1_d + T2_d) \quad (2)$$

The multiplication of two triangular fuzzy numbers

$$< T3 >=< T1 > \otimes < T2 >= (\min(T1_s * T1_d; T1_s * T2_d; T1_d * T2_s; T2_s * T2_d); T1_m * T2_m; \max(T1_s * T1_d; T1_s * T2_d; T2_s * T1_d; T2_s * T2_d)) \quad (3)$$

The subtraction of two triangular fuzzy numbers

$$< T3 >=< T1 > (-) < T2 >= (T1_s - T2_d; T1_m - T2_m; T1_d - T2_a) \quad (4)$$

The division of two triangular fuzzy numbers

$$< T3 >=< T1 > (/) < T2 >= (\min(T1_s / T2_d; T1_s / T1_d;; T2_s * T2_d T1_d * T2_s); T1_m / T2_m; \max(T1_s / T2_d; T1_s / T1_d;; T2_s * T2_d T1_d * T2_s)) \quad (5)$$

Analyzing this trend of increasing the degree of operationalization of fuzzy numbers in different applications and technology transfer to different fields, operations with fuzzy numbers have seen many developments (Maturo, 2009; Tofan, 207)):

The addition $< T3 >=< T1 > \oplus < T2 >= (T1_s + T2_s; T1_m + T2_m; T1_d + T2_d)$ (6)

The multiplication $< T3 >=< T1 > \otimes < T2 >= (T1_s * T2_s; T1_m * T2_m; T1_d * T2_d)$, (for $T1_s, T2_s \geq 0$) (7)

The subtraction $< T3 >=< T1 > (-) < T2 >= (T1_s - T2_d; T1_m - T2_m; T1_d - T2_s)$ (8)

The division $< T3 >=< T1 > (/) < T2 >= (T1_s / T2_d; T1_m / T2_m; T1_d / T2_s)$, (for $T1_s, T2_s \geq 0$) (9)

In order to be able to work with these triangular fuzzy numbers, many simple or synthetic indicators have been defined. Among the simple indicators we recall (Tofan, 2017; Gherasim, 2005):

The core (which coincides with T_m):	$N(T) = \{T_m\}$
The support:	$Sp(T) = (T_s, T_d)$
The length of the support:	$LSP(T) = T_d - T_s \geq 0$
The middle of the core:	$NN(T) = T_m$
The middle of the support:	$SP(T) = \frac{T_s + T_d}{2}$ (10)
Area to the left:	$S^L(T) = \int_{T_s}^{NN(T)} \mu_T(x)dx$
Area to the right:	$S^R(T) = \int_{NN(T)}^{T_s} \mu_T(x)dx$
Total area:	$S(T) = S^L(T) + S^R(T)$
The sign:	$\delta(T) = \begin{cases} \text{sign(NN(T))}, & \text{NN(T)} \neq 0 \\ \text{sign(SP(T))}, & \text{NN(T)} = 0 \end{cases}$

The synthetic indicators are real numbers associated with fuzzy numbers whose value belongs to the support, which are determined by the shape and size of the fuzzy numbers. The purpose of developing these sizes was to build a real synthetic image representative of a triangular fuzzy number, thus helping to ease the specific calculations.

3. Material and methods

A different step from these approaches was developed by Gherasim (2005) who developed an associated indicator called the center of gravity of the fuzzy number:

$$G(T) = \frac{T_s + 2T_m + T_d}{4} \quad (11)$$

Starting from these concepts are proposed (Gherasim, 2005) the following elementary operations:

The addition $<T1> \oplus <T2> = (T1_s + T2_s; T1_m + T2_m; T1_d + T2_d)$ (12)

The multiplication of a triangular fuzzy number with a scalar $a \in R$

$$a \cdot <T1> = (a \cdot T1_s; a \cdot T1_m; a \cdot T1_d) \quad (13)$$

The multiplication $<T1> \otimes <T2> = \frac{<T1> \cdot G(T2) + T2 \cdot G(T1)}{2}$ (14)

The subtraction $<T1>(-)<T2> = (T1_s - T2_s; T1_m - T2_m; T1_d - T2_d$ (15)

The division $<T1>(/)<T2> = \frac{<T1> \cdot G(T2) + <T2> \cdot G(T1)}{2G(T2)^2}$ (16)

To capture the potential of endogenous uncertainty, we propose (where the situation requires it, where the internal entropy of the system cannot be accurately or probabilistically determined) the use of fuzzy numbers with associated variable center of gravity.

The synthetic indicators are real numbers associated with fuzzy numbers whose value belongs to the support, which are determined by the shape and size of the fuzzy numbers. The purpose of developing these sizes was to build a real synthetic image representative of a triangular fuzzy number, thus helping to ease the specific calculations.

We intend to discuss the index of uncertainty management and variability center of gravity associated to a general fuzzy number starting from the general definition (Alecu , 2011, p.7 , Alecu, 2019, p. 23):

$$G(<T>_\alpha) = NN(T) + (\alpha - 1) \cdot S^L(T) + \alpha \cdot S^R(T) \quad (17)$$

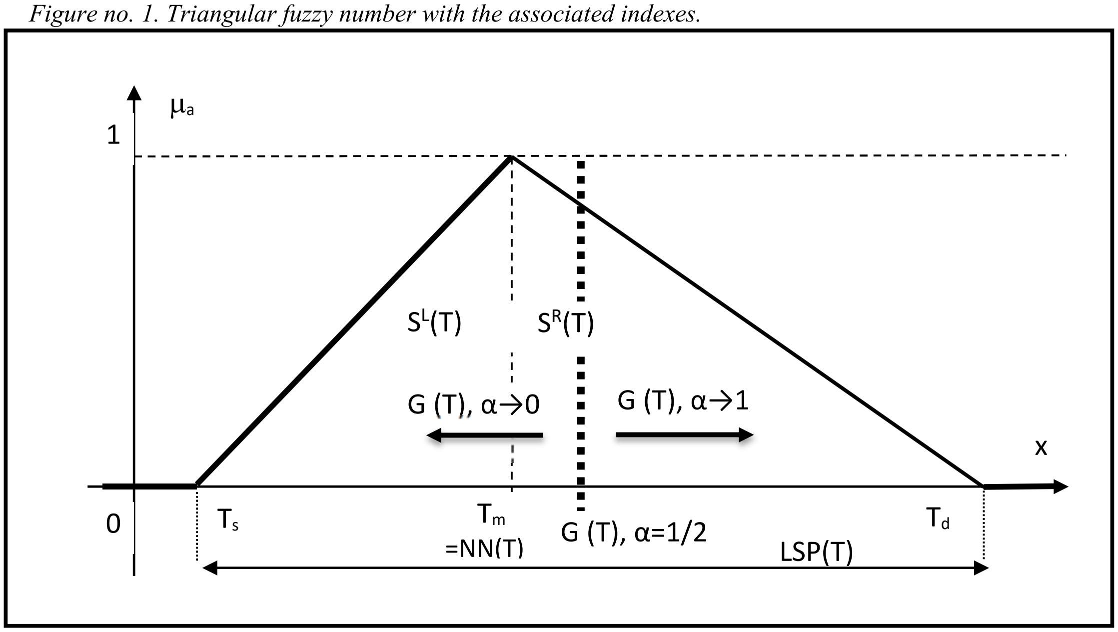
where: $\alpha \in [0,1]$ - is an indicator of absorption / uncertainty management.

The uncertainty is captured by an associated coefficient - α which can take strictly positive and subunit values, between 0 and 1.

As its values experience a tendency toward 0, the fuzzy numbers capture a state of total ambiguity and pessimism internal to the system, which influences the evaluations made. In the diametrically opposite case, when the values of α experience a tendency towards 1, an optimistic state is expressed, where the mechanisms of absorption of uncertainty tend to cover a large part of the difficulties.

The way in which uncertainty is represented and managed true a triangular fuzzy number with variable associated indicator can be seen in the following graph.

Figure no. 1. Triangular fuzzy number with the associated indexes.



Source: Alecu 2019, p. 25

Let there be triangular fuzzy numbers with variable centers of gravity:

$<T1>_\alpha = (T1_s, T1_m, T1_d)_\alpha$, $<T2>_\beta = (T2_s, T2_m, T2_d)_\beta$ and $<T3>_\gamma = (T3_s, T3_m, T3_d)_\gamma \in$ NFT$_r$, where $\alpha, \beta, \gamma \in [0,1]$ are the assumed levels of absorption of uncertainty, then the specific gravity centers with respectively are $G(T1_\alpha)$, $G(T2_\beta)$, $G(T3_\gamma)$.

Elementary operations with triangular fuzzy numbers using associated indicators are poposed Alecu (2016, 2019)]:

The addition

$$<T3>_\gamma = <T1>_\alpha \oplus <T2>_\beta = \begin{cases} (T1_s + T2_s; T1_m + T2_m; T1_d + T2_d)_\gamma \\ \gamma = (\alpha(T1_s + T2_s) + \beta(T1_d + T2_d))/(T1_s + T2_s + T1_d + T2_d) \end{cases} \quad (18)$$

The multiplication of a triangular fuzzy number with a scalar $a \in R$

$$<T3>_\gamma = a \cdot <T1>_\alpha = \begin{cases} (a \cdot T1_s; a \cdot T1_m; a \cdot T1_d)_\gamma, & \gamma = \alpha, t \ge 0 \\ (a \cdot T1_d; a \cdot T1_m; a \cdot T1_s)_\gamma, & \gamma = 1 - \alpha, t < 0 \end{cases} \quad (19)$$

The multiplication of two triangular fuzzy numbers

$$<T3>_\gamma = <T1>_\alpha \otimes <T2>_\beta =$$
$$= \begin{cases} (\frac{T1_s G(T2)_\beta + G(T1)_\alpha T2_s}{2}; \frac{T1_m G(T2)_\beta + G(T1)_\alpha T2_m}{2}; \frac{T1_d G(T2)_\beta + G(T1)_\alpha T2_d}{2} \\ \gamma = (\alpha \cdot \beta)/((\alpha + \beta)/2) \end{cases} \quad (20)$$

The subtraction

$$<T3>_\gamma = <T1>_\alpha (-) <T2>_\beta = \begin{cases} (T1_s - T2_s; T1_m - T2_m; T1_d - T2_d)_\gamma \\ \gamma = (\alpha(T1_s + T2_s) + (1-\beta)(T1_d + T2_d))/(T1_s + T2_s + T1_d + T2_d) \end{cases} \quad (21)$$

The division

$$<T3>_\gamma = <T1>_\alpha (/) <T2>_\beta = \begin{cases} \frac{<T1>_\alpha \cdot G(<T2>_\beta) + <T2>_\beta \cdot G(<T1>_\alpha)}{2G(<T2>_\beta)^2} \\ \gamma = (\alpha \cdot (1-\beta))/((\alpha + (1-\beta)/2) \end{cases} \quad (22)$$

The ordering of the triangular fuzzy numbers with variable centers of gravity is done upon the basis of several successive criteria: the gravity center criterion, the core means criterion, the criterion of the sign lengths of the supports (cores).

4. Findings

In order to highlight the way in which these elements can be used in uncertainty management, we propose the analysis for different values of the variable center of gravity (α = 0, 1/2, 1) the optimal variants that can be identified by the Wald criterion, max- max and Laplace.

We discuss the following decision-making situation for setting up a water supply point for a local company. Several decisional situations (V1-V5) and 4 states of nature (E1-E4) were are taken into account (depending on technical, technological, financial and socio-economic indicators), obtaining the following matrix of consequences.

Table 1 Consequence matrix

	V1	V2	V3	V4	V5
E1	[0.33; 0.44; 0.77]	[0.11; 0.22; 0.44]	[0.11; 0.55; 0.77]	[0.22; 0.33; 0.55]	[0.11; 0.44; 0.55]
E2	[0.44; 0.66; 0.77]	[0.33; 0.55; 0.88]	[0.44; 0.55; 0.66]	[0.22; 0.44; 0.66]	[0.44; 0.77; 0.88]
E3	[0.33; 0.55; 0.88]	[0.44; 0.55; 0.77]	[0.33; 0.55; 0.66]	[0.44; 0.88; 0.99]	[0.33; 0.66; 0.99]
E4	[0.55; 0.66; 0.77]	[0.33; 0.66; 0.88]	[0.44; 0.55; 0.88]	[0.55; 0.77; 0.99]	[0.44; 0.66; 0.88]

Source: data obtained by the authors are normalized from the decision situation

By applying the Wald criterion, the following optimal variants were obtained:
α = 1/2, V3> V1> V5 > V4> V2
α = 0, V1> V3> V4> V5> V2
α = 1, V1> V3> V4> V5> V2
By applying the max-max criterion, the following optimal variants were identified>
α = 1/2, V4> V5> V1> V2> V3
α = 0, V4> V1> V5> V3> V2
α = 1, V4> V5> V2> V1> V3
By applying the Laplace criterion with equidistant probabilities, the following optimal variants were obtained.
α = 1/2, V5>V4>V1>V3>V2
α = 0, V1> V5>V4>V3>V2
α = 1, V5>V4>V1>V3>V2
We notice that by managing the uncertainty using different strategies and representing with the help of the uncertainty absorption indicator α (in our case for the 3 hypothetical situations 1, 1/2 respectively 1) the manager can identify different optimal decision variants. Therefore, the way in which the manager approaches the uncertainty in a certain decision-making context can be represented through this theoretical construct.

5. Conclusions

Analysing the contributions to the management of uncertainty in social and economic processes, from an epistemological point of view we can say that the use of fuzzy numbers is a great progress. As for assessing the development of elementary processes by using variable indicators as associated to fuzzy numbers, in addition to an interdisciplinary approach of decision management, results show an important addition for the interdisciplinary socio-economic phenomena, of the governing law and the holistic uncertainty of the risks for their development, but also within strategy performance and implemented programs reviews.

In the case of our article, the concern was to develop a new model for substantiating decisions in conditions of persistent degree III uncertainty starting from the classical methods of uncertainty management, formal or informal, using an interdisciplinary theoretical framework based on numbers. triangular fuzzy with variable associated indicators, with specific elementary operations, with rules and criteria for managing inaccuracy and imperfection of information, etc ..

Reflections on the specificities of for developing a variable center of gravity vastly associated to fuzzy numbers may suggest: the use of estimated information specific for uncertain events that are insufficiently statistically analyzed (seen as possibilities and not possible events); another point of view takes into consideration environmental tendencies (growth, recession etc.); one other view overtakes potential informational entropies coming from internal system assessments; one contribution avoids the excess use of mathematical expectation; it may also define the affinity and/or aversion towards risk of the assessor; the last point of view that is considered lens towards the subjectivity of the absorption mechanisms specific for the uncertainty, as defined by organizational management.

This construct of fuzzy numbers with associated variable indicators and specific elementary operations becomes an important tool for managing uncertainty in management methods. Through an interdisciplinary approach aimed to achieving the proposed objectives, a logically structured model can be defined in assuming one direction of action to the detriment of another based on medium and long-term strategies.

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