Linear Fuzzy Regressions versus Power Fuzzy Regressions

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Abstract

Under the current conditions, the input data (values of the point cloud coordinates) necessary to achieve correlations and regressions between two variables representing two economic indicators are highly uncertain.

However, by consulting a consistent number of specialists (a minimum of 3), by ordering and grouping the values obtained from them in 3 groups we can obtain, as arithmetic mean of the groups, modeling of the input data with triangular fuzzy numbers.

In the article we presented the working method for obtaining linear fuzzy regressions and power fuzzy regressions then we compared the two types of fuzzy regressions by calculating the associated correlation coefficients.

Key words: linear fuzzy regressions, fuzzy power regressions, triangular fuzzy numbers
J.E.L. classification: C40, D80

1. Introduction

The modeling in a variable economic and social context is at a standstill because nowadays the uncertainty of information has exceeded all the forecasts of specialists.

The input data for the application of classical methods for solving various economic models are difficult to estimate.

This requires the use of fuzzy numbers to model the uncertainty of the data.

And when we try to determine the existence of correlations and regressions between two economic and social indicators, we encounter the same shortcomings.

The aim of the article is to draw the way for obtaining linear and power fuzzy regression functions by using triangular fuzzy numbers and their specific operations.

2. Literature review

In order to achieve the objective proposed in the article, theoretical notions from two fields are needed: linear and power type regressions as well as triangular fuzzy numbers and operations with them.

The specialized literature on the field of regressions is particularly rich and is centuries old.

Just as an example we mention here chronologically, the following works: (Ţarcă, 1979), (Jaba, 1986), (Gromîco, 1990), (Baron *et al*, 1996), (Biji *et al*, 2000) etc.

Even if the fuzzy theory is only a few decades old (over half a century) the specialized literature that has it as its main subject is just as rich and very topical.

Also as an example we list here the following works: (Zadeh, 1965) believed as the first work of fuzzy theory, (Kaufmann, 1973), (Negoiţă *et al*, 1974), (Tacu *et al*, 1994), (Bojadziev *et al*, 1995) etc.

Another view on operations and functions with fuzzy numbers is described in the works of (Gherasim, 2005), (Gherasim, 2008), (Alecu, 2011), (Alecu *et al*, 2021) and so on.

This view consists in the much more restrictive definition of operations with fuzzy numbers but has great advantages by preserving the associated center of gravity and preserving the type of operators.

Thus the operations with triangular fuzzy numbers also result in triangular fuzzy numbers.

In this article, triangular fuzzy numbers allow the design of an algorithm for obtaining linear and power fuzzy regressions.

3. Research methodology

For the use of triangular fuzzy numbers in the article, we briefly present the basics of operations with them.

A fuzzy triangular number is an ordered triplet of real numbers:

$\tilde{u} = (u_1, u_2, u_3) \quad , \; u_1 \le u_2 \le u_3$ (1)

The center of gravity, the multiplication with a real scalar, the addition, the subtraction, the multiplication, the division and order with triangular fuzzy numbers are found in (Gherasim, 2008, p.110) and are defined by the following relations:

$< \tilde{u} > = \dfrac{u_1 + 2 \cdot u_2 + u_3}{4} \in \mathbf{R}$ (2) the associated center of gravity

$t \cdot < \tilde{u} > = \begin{cases} (t \cdot u_1, t \cdot u_2, t \cdot u_3) \; , t \ge 0 \\ t \cdot u_3, t \cdot u_2, t \cdot u_1) \; , t < 0 \end{cases} \; , \; t \in \mathbf{R}$ (3) the multiplication with a real scalar

$\tilde{u} + \tilde{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$ (4) the addition

$\tilde{u} - \tilde{v} = (u_1 - v_3, u_2 - v_2, u_3 - v_1)$ (5) the subtraction

$\tilde{u} \cdot \tilde{v} = \dfrac{< \tilde{u} > \cdot \tilde{v} + < \tilde{v} > \cdot \tilde{u}}{2}$ (6) the multiplication

$\dfrac{\tilde{u}}{\tilde{v}} = \dfrac{\tilde{u} \cdot \tilde{v}}{< \tilde{v} >^2} = \dfrac{< \tilde{u} > \cdot \tilde{v} + < \tilde{v} > \cdot \tilde{u}}{2 \cdot < \tilde{v} >^2}$ (7) the division

$< \tilde{u} > \; < \; < \tilde{v} > \quad \Rightarrow \quad \tilde{u} \prec \tilde{v}$ (8) the order relation (first criterion)

The article also uses some fuzzy variable functions that are calculated with the following relation (Gherasim, 2008, pp.29-30):

$f(\tilde{u}) = \dfrac{f(< \tilde{u} >)}{< \tilde{u} >} \cdot \tilde{u}$ (9) functions of variable fuzzy number

To clarify how to work with fuzzy triangular numbers we will provide the following examples:

$$< (2,7,8) > = \frac{2 + 2 \cdot 7 + 8}{4} = \frac{24}{4} = 6 \quad \Rightarrow \quad (2,7,8)_6 \qquad (1,2,3)_2$$

$$2 \cdot (1,2,3)_2 = (2 \cdot 1, 2 \cdot 2, 2 \cdot 3) = (2,4,6)_4, \qquad (-2) \cdot (1,2,3)_2 = (-2 \cdot 3, -2 \cdot 2, -2 \cdot 1) = (-6,-4,-2)_{-4}$$

$$\frac{(1,2,3)_2}{2} = \frac{1}{2} \cdot (1,2,3)_2 = (\frac{1}{2} \cdot 1, \frac{1}{2} \cdot 2, \frac{1}{2} \cdot 3) = (0.5,\ 1,\ 1.5)_2$$

$$(2,7,8)_6 + (1,2,3)_2 = (2+1, 7+2, 8+3) = (3, 9, 11)_8$$

$$(2,7,8)_6 - (1,2,3)_2 = (2-3, 7-2, 8-1) = (-1, 5, 7)_4$$

$$(2,7,8)_6 \cdot (1,2,3)_2 = \frac{6 \cdot (1,2,3) + 2 \cdot (2,7,8)}{2} = \frac{(6+4, 12+14, 18+16)}{2} = (5, 13, 17)_{12}$$

$$\frac{(2,7,8)_6}{(1,2,3)_2} = \frac{(2,7,8)_6 \cdot (1,2,3)_2}{2^2} = \frac{6 \cdot (1,2,3) + 2 \cdot (2,7,8)}{2 \cdot 4} = \frac{(5,13,17)}{4} = (1.25, 3.25, 4.25)_3$$

$$6 > 2 \quad \Rightarrow \quad (2,7,8)_6 \succ (1,2,3)_2$$

$$\ln(2,4,6)_4 = \frac{\ln 4}{4} \cdot (2,4,6) \approx \frac{1.386}{4} \cdot (2,4,6) \approx 0.346 \cdot (2, 4, 6) = (0.693, 1.386, 2.079)_{1.386}$$

$$e^{(1,2,3)_2} = \frac{e^2}{2} \cdot (1,2,3)) \approx 3.6945 \cdot (1,2,3) = (3.695, 7.389, 11.084)_{7.389}$$

$$(2,4,6)_4^{(1,2,3)_2} = \frac{4^2}{4} \cdot (2,4,6) = (8,16,24)_{16}, \qquad \sqrt{(2,4,6)_4} = \frac{\sqrt{4}}{4} \cdot (2,4,6) = \frac{(2,4,6)}{\sqrt{4}} = (1,2,3)_2 .$$

To establish a linear regression $y = a \cdot x + b$ between the independent variable x and the dependent variable y, consider a cloud composed of n points $\{P_k(x_k, y_k)\}_{k=\overline{1,n}}$.

The right line for which the sum of the squares of the distances from the cloud points to the right is minimal is obtained by determining the parameters a and b that define it.

The relations by which the two parameters are calculated are found in many works of which we give the following example (Jaba, 1986, pp.207-209), (Gromîco, 1990, pp.135-137):

$$a = \frac{n \cdot S_{xy} - S_x \cdot S_y}{n \cdot S_{xx} - S_x \cdot S_x}, \qquad b = \frac{S_{xx} \cdot S_y - S_x \cdot S_{xy}}{n \cdot S_{xx} - S_x \cdot S_x} \qquad (10)$$

where:

$$S_x = \sum_{k=1}^{n} x_k, \quad S_y = \sum_{k=1}^{n} y_k, \quad S_{xx} = \sum_{k=1}^{n} (x_k \cdot x_k), \quad S_{xy} = \sum_{k=1}^{n} (x_k \cdot y_k) \qquad (11)$$

In order to establish a power type regression $y = q \cdot x^p$, one can notice that by logarithm we also obtain a linear relation:

$$y = q \cdot x^p \overset{\ln}{\Leftrightarrow} \ln y = p \cdot \ln x + \ln q \Leftrightarrow \hat{y} = p \cdot \hat{x} + \hat{q} \quad \text{,where:} \quad \ln w \overset{not.}{=} \hat{w} .$$

Due to the fact above, the relations for the calculation of the two defining parameters p and q are deduced from the relations (10) – (11), for example in the paper (Ţarcă, 1979, pp.106-108):

$$p = \frac{n \cdot S_{\hat{x}\hat{y}} - S_{\hat{x}} \cdot S_{\hat{y}}}{n \cdot S_{\hat{x}\hat{x}} - S_{\hat{x}} \cdot S_{\hat{x}}}, \qquad \hat{q} = \ln q = \frac{S_{\hat{x}\hat{x}} \cdot S_{\hat{y}} - S_{\hat{x}} \cdot S_{\hat{x}\hat{y}}}{n \cdot S_{\hat{x}\hat{x}} - S_{\hat{x}} \cdot S_{\hat{x}}}, \qquad q = e^{\hat{q}} \qquad (12)$$

where:

$$S_{\hat{x}} = \sum_{k=1}^{n} \hat{x}_k, \quad S_{\hat{y}} = \sum_{k=1}^{n} \hat{y}_k, \quad S_{\hat{x}\hat{x}} = \sum_{k=1}^{n} (\hat{x}_k \cdot \hat{x}_k), \quad S_{\hat{x}\hat{y}} = \sum_{k=1}^{n} (\hat{x}_k \cdot \hat{y}_k) \qquad (13)$$

In order to establish which of the two regressions is more appropriate, we will associate to each the Pearson correlation coefficient (Baron *et al*, 1996, p.173), (Biji *et al*, 2000, pp.306-307) and so forth:

$$R_{Lin} = \sqrt{1 - \frac{\sum_{k=1}^{n} \left(y_k - y_{(x_k)}^{(lin)}\right)^2}{\sum_{k=1}^{n} \left(y_k - \overline{y}\right)^2}}, \qquad R_{Ptr} = \sqrt{1 - \frac{\sum_{k=1}^{n} \left(y_k - y_{(x_k)}^{(Ptr)}\right)^2}{\sum_{k=1}^{n} \left(y_k - \overline{y}\right)^2}} \qquad (14)$$

Where $\overline{y}$ is the average of the values of the dependent variable in the point cloud:

$$\overline{y} = \frac{S_y}{n} = \frac{\sum_{k=1}^{n} y_k}{n} \qquad (15)$$

The most appropriate regression is the one with the R value closest to 1 (highest R).

The classical methods such as the method described above can no longer be applied in conditions of increased uncertainty in a variable economic and social environment.

Thus, the coordinates of the cloud points have a high degree of uncertainty.

For this reason it is necessary to consult a large number of specialists ($s \geq 3$), from which to obtain s values for each coordinate of the cloud points.

After ordering the s values we will choose the first (with the lowest value) as the first component of that coordinate, the last (with the highest value) as the third component and the average of the other s–2 values as the second component of that coordinate:

$$x_1^{(k)} = \min_{i=\overline{1,s}} v_i = v_1, \quad x_3^{(k)} = \max_{i=\overline{1,s}} v_i = v_s, \quad x_2^{(k)} = \frac{\sum_{i=2}^{s-1} v_i}{s-2}, \quad \tilde{x}^k = (x_1^{(k)}, x_2^{(k)}, x_3^{(k)}) \qquad (16)$$

The described procedure must be applied to all the coordinates of the points in the cloud.

After having performed all the operations, a cloud composed of n fuzzy points is obtained:

$$\left\{\tilde{P}_k(\tilde{x}^{(k)}, \tilde{y}^{(k)})\right\}_{k=\overline{1,n}} \quad (17)$$

Now one can apply the algorithm described in the relations (10) – (15) by which one can obtain linear and power fuzzy regressions:

$$\tilde{y} = \tilde{a} \cdot \tilde{x} + \tilde{b} \quad \text{Namely} \quad \tilde{y} = \tilde{q} \cdot \tilde{x}^{\tilde{p}} \quad (18)$$

All the necessary calculations are performed according to the operations with triangular fuzzy numbers described in detail in the relations (1) – (12).

4. Findings

Next we will try to clarify the operating mode in order to obtain the two types of fuzzy regressions through a hypothetical case study.

Thus, with the help of our own software, dedicated to this article, we randomly generated the coordinates of a cloud $\left\{\tilde{P}_k(\tilde{x}^{(k)}, \tilde{y}^{(k)})\right\}_{k=\overline{1,n}}$ made of n=30 fuzzy points.

The generated coordinates are the triangular fuzzy numbers presented in the following table:

Table no. 1. The coordinates of the cloud composed of n = 30 fuzzy points

k	$\tilde{x}^{(k)}$	$\tilde{y}^{(k)}$	k	$\tilde{x}^{(k)}$	$\tilde{y}^{(k)}$
1	$(7, 10, 13)_{10}$	$(5, 6, 8)_{6.25}$	16	$(30, 34, 59)_{39.25}$	$(156, 215, 285)_{217.75}$
2	$(7, 10, 16)_{10.75}$	$(7, 10, 13)_{10}$	17	$(22, 47, 54)_{42.5}$	$(114, 233, 329)_{227.25}$
3	$(9, 13, 17)_{13}$	$(8, 14, 20)_{14}$	18	$(26, 46, 66)_{46}$	$(190, 244, 329)_{251.75}$
4	$(8, 16, 24)_{16}$	$(15, 23, 27)_{22}$	19	$(36, 51, 62)_{50}$	$(207, 313, 350)_{295.75}$
5	$(10, 21, 23)_{18.75}$	$(19, 28, 40)_{28.75}$	20	$(34, 55, 68)_{53}$	$(211, 303, 479)_{324}$
6	$(16, 17, 24)_{18.5}$	$(25, 36, 44)_{35.25}$	21	$(36, 51, 68)_{51.5}$	$(177, 389, 495)_{362.5}$
7	$(11, 21, 31)_{21}$	$(26, 47, 55)_{43.75}$	22	$(31, 59, 75)_{56}$	$(261, 413, 472)_{389.75}$
8	$(19, 21, 32)_{23.25}$	$(35, 59, 74)_{56.75}$	23	$(39, 61, 69)_{57.5}$	$(227, 433, 596)_{422.25}$
9	$(20, 24, 38)_{26.5}$	$(45, 63, 85)_{64}$	24	$(32, 59, 83)_{58.25}$	$(244, 455, 587)_{435.25}$
10	$(18, 26, 34)_{26}$	$(41, 90, 101)_{80.5}$	25	$(39, 68, 78)_{63.25}$	$(297, 487, 707)_{494.5}$
11	$(19, 29, 38)_{28.75}$	$(60, 105, 140)_{102.5}$	26	$(47, 53, 89)_{60.5}$	$(351, 535, 714)_{533.75}$
12	$(23, 36, 40)_{33.75}$	$(80, 116, 171)_{120.75}$	27	$(47, 58, 88)_{62.75}$	$(339, 593, 757)_{570.5}$
13	$(24, 36, 48)_{36}$	$(105, 140, 182)_{141.75}$	28	$(39, 60, 83)_{60.5}$	$(367, 708, 826)_{652.25}$
14	$(23, 35, 44)_{34.25}$	$(117, 156, 210)_{159.75}$	29	$(38, 53, 98)_{60.5}$	$(382, 658, 863)_{640.25}$
15	$(24, 36, 52)_{37}$	$(139, 174, 215)_{175.5}$	30	$(34, 65, 99)_{65.75}$	$(572, 824, 946)_{791.5}$

Source: randomly generated data on the computer

In order to obtain the linear fuzzy regression we will calculate the four fuzzy amounts with the relations (11) and their results will be replaced in the relations (10).

We mention that all the calculations were made with our own dedicated software with a large number of decimals, made by the author in the Microsoft Visual FoxPro programming language.

The results listed below in the article are rounded to 2–3 decimals for those with modules less than 0.1 or even whole numbers for those with modules in the hundreds and over.

$$\tilde{S}_{\tilde{x}} = \sum_{k=1}^{n} \tilde{x}^{(k)} = (768,\ 1171,\ 1613)_{1181}, \qquad \tilde{S}_{\tilde{y}} = \sum_{k=1}^{n} \tilde{y}^{(k)} = (4822,\ 7870,\ 10120)_{7671}$$

The products of the form $\tilde{x}^{(k)} \cdot \tilde{x}^{(k)}$ and $\tilde{x}^{(k)} \cdot \tilde{y}^{(k)}$ were calculated according to relations (6) and are listed in the table below:

Table no. 2. The product of the form $\widetilde{x}^{(k)} \cdot \widetilde{x}^{(k)}$ *and* $\widetilde{x}^{(k)} \cdot \widetilde{y}^{(k)}$

k	$\widetilde{x}^{(k)} \cdot \widetilde{x}^{(k)}$	$\widetilde{x}^{(k)} \cdot \widetilde{y}^{(k)}$
1	$(70, 100, 130)_{100}$	$(46.9, 61.3, 80.6)_{62.5}$
2	$(75.25, 107.5, 172)_{115.6}$	$(72.6, 103.8, 149.9)_{107.5}$
…	…	…
30	$(2236, 4274, 6509)_{4323}$	$(32260, 52813, 70279)_{52041}$

Source: the author's calculations

The amounts of these products on the two columns are as follows:

$$\widetilde{S}_{\widetilde{x}\widetilde{x}} = \sum_{k=1}^{n}\left(\widetilde{x}^{(k)} \cdot \widetilde{x}^{(k)}\right) \approx (36167, 55503, 76825)_{55999}$$

$$\widetilde{S}_{\widetilde{x}\widetilde{y}} = \sum_{k=1}^{n}\left(\widetilde{x}^{(k)} \cdot \widetilde{y}^{(k)}\right) \approx (261708, 416024, 560203)_{413490}$$

The parameters $\widetilde{a}$ and $\widetilde{b}$ that define the linear fuzzy regression $\widetilde{y} = \widetilde{a} \cdot \widetilde{x} + \widetilde{b}$ are calculated according to the relations (10), as follows:

$$\widetilde{a} = \frac{n \cdot \widetilde{S}_{\widetilde{x}\widetilde{y}} - \widetilde{S}_{\widetilde{x}} \cdot \widetilde{S}_{\widetilde{y}}}{n \cdot \widetilde{S}_{\widetilde{x}\widetilde{x}} - \widetilde{S}_{\widetilde{x}} \cdot \widetilde{S}_{\widetilde{x}}} = \frac{(-4309602,\ 3343399,\ 11013834)_{3347757}}{(-819532,\ 282417,\ 1397919)_{285805}} \approx (-24.3,\ 11.6,\ 47.9)_{11.7}$$

$$\widetilde{b} = \frac{\widetilde{S}_{\widetilde{X}\widetilde{X}} \cdot \widetilde{S}_{\widetilde{Y}} - \widetilde{S}_{\widetilde{X}} \cdot \widetilde{S}_{\widetilde{X}\widetilde{Y}}}{n \cdot \widetilde{S}_{\widetilde{X}\widetilde{X}} - \widetilde{S}_{\widetilde{X}} \cdot \widetilde{S}_{\widetilde{X}}} = \frac{(-390485119, -54485946, 264710811)_{-58686550}}{(-819532,\ 282417,\ 1397919)_{285805}} \approx (-389, -197, -39)_{-205}$$

The linear fuzzy regression is of the following form:

$$\boxed{\widetilde{y} = (-24.3,\ 11.6,\ 47.9)_{11.7} \cdot \widetilde{x} - (39, 197, 389)_{205}}$$

In order to obtain the fuzzy power type regression, it is necessary to first perform some logarithmic operations performed according to relation (9), then the achievements of products and amounts of the logarithms.

For instance: $\ln \widetilde{x}^{(1)} = \ln(7,10,13)_{10} = \frac{\ln 10}{10} \cdot (7,10,13)_{10} \approx (1.6, 2.3, 3.0)_{2.3}$

Table no. 3 The logarithmization of triangular fuzzy numbers and products of logarithms

k	$\ln \widetilde{x}^{(k)}$	$\ln \widetilde{y}^{(k)}$	k	$\ln \widetilde{x}^{(k)} . \ln \widetilde{x}^{(k)}$	$\ln \widetilde{x}^{(k)} . \ln \widetilde{y}^{(k)}$
1	$(1.6, 2.3, 3.0)_{2.3}$	$(1.5, 1.8, 2.3)_{1.8}$	1	$(3.7, 5.3, 6.9)_{5.3}$	$(3.2, 4.1, 5.4)_{4.2}$
2	$(1.5, 2.2, 3.5)_{2.4}$	$(1.6, 2.3, 3.0)_{2.3}$	2	$(3.7, 5.2, 8.4)_{5.6}$	$(3.7, 5.3, 7.6)_{5.5}$
…	…	…	…	…	…
30	$(2.2, 4.1, 6.3)_{4.2}$	$(4.8, 6.9, 8.0)_{6.7}$	30	$(9.1, 17.3, 26.4)_{17.5}$	$(17.3, 28.4, 37.7)_{27.9}$

Source: the author's calculations

The results of the sums with logarithms and with the products of logarithms calculated according to the relations (13) are the following:

$$\widetilde{S}_{\hat{\widetilde{x}}} = \sum_{k=1}^{n}\left(\ln \widetilde{x}^{(k)}\right) = (69.5, 105, 144.7)_{106}, \qquad \widetilde{S}_{\hat{\widetilde{y}}} = \sum_{k=1}^{n}\left(\ln \widetilde{y}^{(k)}\right) = (95, 151, 196)_{148}$$

$$\widetilde{S}_{\hat{\widetilde{x}}\hat{\widetilde{x}}} = \sum_{k=1}^{n}\left(\ln \widetilde{x}^{(k)} \cdot \ln \widetilde{x}^{(k)}\right) = (251, 382, 525)_{385}, \quad \widetilde{S}_{\hat{\widetilde{x}}\hat{\widetilde{y}}} = \sum_{k=1}^{n}\left(\ln \widetilde{x}^{(k)} \cdot \ln \widetilde{y}^{(k)}\right) = (352, 549, 734)_{546}.$$

The parameters $\widetilde{p}$ and $\widetilde{q}$ that define the power fuzzy regression $\widetilde{y} = \widetilde{q} \cdot \widetilde{x}^{\widetilde{p}}$ are calculated similarly to the operations performed at the linear fuzzy regression, with the relations (12), as follows:

$$\tilde{p} = \frac{n \cdot \tilde{S}_{\hat{x}\hat{y}} - \tilde{S}_{\hat{x}} \cdot \tilde{S}_{\hat{y}}}{n \cdot \tilde{S}_{\hat{x}\hat{x}} - \tilde{S}_{\hat{x}} \cdot \tilde{S}_{\hat{x}}} = \frac{(-10533,\ 668,\ 11836)_{660}}{(-7823,\ 279,\ 8377)_{278}} \approx (-52,\ 2.4,\ 57)_{2.4}$$

$$\hat{\tilde{q}} = \ln \tilde{q} = \frac{\tilde{S}_{\hat{x}\hat{x}} \cdot \tilde{S}_{\hat{y}} - \tilde{S}_{\hat{x}} \cdot \tilde{S}_{\hat{x}\hat{y}}}{n \cdot \tilde{S}_{\hat{x}\hat{x}} - \tilde{S}_{\hat{x}} \cdot \tilde{S}_{\hat{x}}} = \frac{(-41601, -558, 38868)_{-962}}{(-7823, 279, 8377)_{278}} \approx (-26,\ -2.7,\ 17.8)_{-3.5}$$

$$\tilde{q} = e^{\hat{\tilde{q}}} \overset{(9)}{=} \frac{e^{<\hat{\tilde{q}}>}}{<\hat{\tilde{q}}>} \cdot \hat{\tilde{q}} = \frac{e^{-3.5}}{-3.5} \cdot (-26,\ -2.7,\ 17.8)_{-3.5}$$

$$\tilde{q} \approx -0.0086 \cdot (-26,\ -2.7,\ 17.8)_{-3.5} \approx (-0.15, 0.023, 0.224)_{0.03}$$

The fuzzy power regression is of the following form:

$$\boxed{\tilde{y} = (-0.15, 0.023, 0.224)_{0.03} \cdot \tilde{x}^{(-52, 2.4, 57)_{2.4}}}$$

In order to compare the two fuzzy regressions and establish the form of the most appropriate regression model, we will make some calculations beforehand.

The average fuzzy value is:

$$\bar{\tilde{y}} = \frac{\tilde{S}_{\tilde{y}}}{n} = \frac{1}{30} \cdot (4822,\ 7870,\ 10120)_{7671} \approx (161,\ 262,\ 337)_{256}.$$

By applying the two regressions

$$\left.\begin{array}{l} \tilde{y}^{(k)}_{Lin} = (-24.3, 11.6, 47.9)_{11.7} \cdot \tilde{x}^{(k)} - (39, 197, 389)_{205} \\ \tilde{y}^{(k)}_{Ptr} = (-0.15,\ 0.023,\ 0.224)_{0.03} \cdot \tilde{x}^{(k)(-52,\ 2.4,\ 57)_{2.4}} \end{array}\right| ,k = \overline{1,n}$$

the following fuzzy values are obtained for the dependent variable:

Table no. 4 Values (triangular fuzzy numbers) obtained by applying fuzzy regressions

k	$\tilde{y}^{(k)}_{Lin}$	$\tilde{y}^{(k)}_{Ptr}$
1	$(-469, -80, 277)_{-88}$	$(-16.4, 6.6, 32.7)_{7.4}$
2	$(-479, -76, 312)_{-79.4}$	$(-19.7, 7.6, 39.6)_{8.8}$
…	…	…
30	$(-990, 566, 2116)_{565}$	$(-1484, 572, 2910)_{643}$

Source: the author's calculations

The fuzzy sums of the squares of the differences are as follows:

$$\tilde{S}_{Med} = \sum_{k=1}^{n} \left(\tilde{y}^{(k)} - \bar{\tilde{y}}\right)^2 = (310584, 1542184, 2531132)_{1481521}$$

$$\tilde{S}_{Lin} = \sum_{k=1}^{n} \left(\tilde{y}^{(k)} - \tilde{y}^{(k)}_{Lin}\right)^2 = (-2043799, 207177, 2327033)_{174397}$$

$$\tilde{S}_{Ptr} = \sum_{k=1}^{n} \left(\tilde{y}^{(k)} - \tilde{y}^{(k)}_{Lin}\right)^2 = (-1474920, 102190, 1565087)_{73637}$$

The correlation coefficients corresponding to the two fuzzy regressions are:

$$\tilde{R}_{Lin} = \sqrt{(1,1,1)_1 - \frac{\tilde{S}_{Lin}}{\tilde{S}_{Med}}} = \sqrt{(0.11, 0.87, 1.68)_{0.88}} = (0.12,\ 0.93,\ 1.79)_{0.94}$$

$$\tilde{R}_{Ptr} = \sqrt{(1,1,1)_1 - \frac{\tilde{S}_{Ptr}}{\tilde{S}_{Med}}} = \sqrt{(0.429, 0.940, 1.493)_{0.950}} = (0.44, 0.964,\ 1.531)_{0.98}$$

Even if the centers of gravity ($0.94 < 0.98$) are both close to 1 and suggest that the two fuzzy correlations are strong, the fuzzy power regression is the most appropriate because its correlation coefficient has a higher center of gravity.

5. Conclusions

In order to establish the validity and credibility of the method for performing fuzzy regressions, we studied the results obtained in terms of reducing them to the associated centers of gravity.

Thus, the two fuzzy regressions now become classical regressions in real numbers:

$$\left.\begin{array}{l} < \tilde{y}^{(k)}_{Lin} > = 11.7 \cdot < \tilde{x}^{(k)} > -205 \\ < \tilde{y}^{(k)}_{Ptr} > = 0.03 \cdot < \tilde{x}^{(k)} >^{2.4} \end{array}\right| , k = \overline{1, n}$$

By applying these two regressions the following values are obtained for the dependent variable:

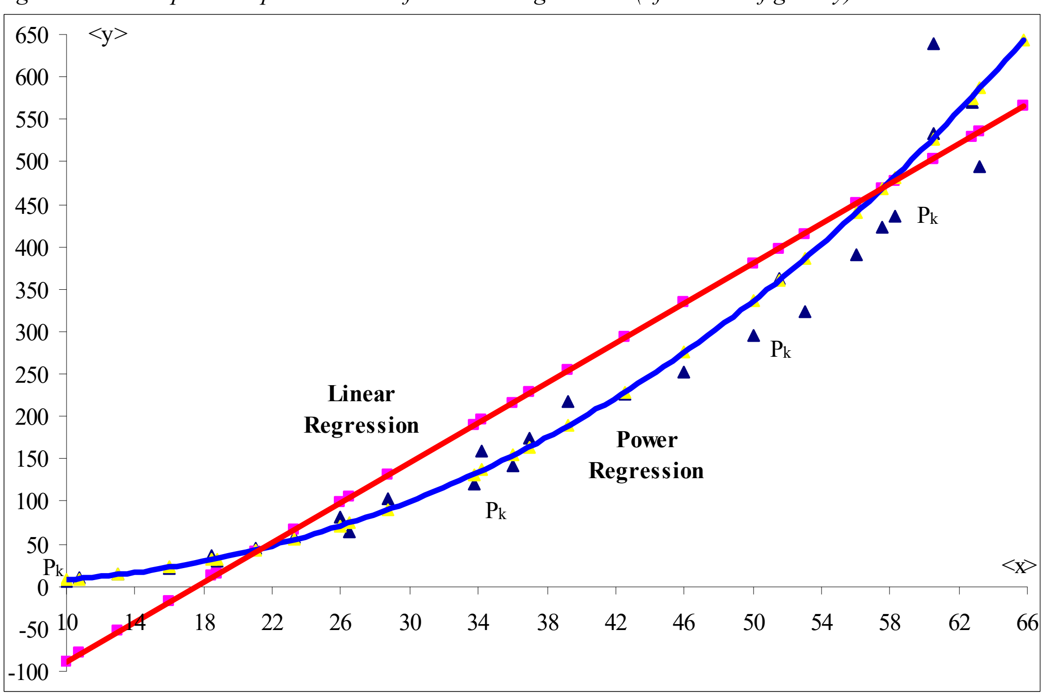
Table no. 5 Values obtained by applying classical regressions

k	$< \tilde{x}^{(k)} >$	$< \tilde{y}^{(k)} >$	$< \tilde{y}^{(k)}_{Lin} >$	$< \tilde{y}^{(k)}_{Ptr} >$
1	10	6.25	–88	7.39
2	10.75	10	–79.4	8.8
3	13	14	–53	13.8
…	…	…	…	…
30	65.75	791.50	565	643

Source: the author's calculations

The graphical representation of the cloud of points formed by the centers of gravity and of the two curves $< \tilde{y}^{(k)}_{Lin} >$ and $< \tilde{y}^{(k)}_{Ptr} >$ is presented as follows:

Figure no. 1. Graphical representation of classical regressions (of centers of gravity)



Source: graphs made by the author

The two curves $y_{(x)} = 11.7 \cdot x - 205$ and $y_{(x)} = 0.03 \cdot x^{2.4}$ allow a pragmatic understanding of the analysis of the linear fuzzy regression function versus the power fuzzy regression function.

By developing the conceptual framework for the uncertain representation of data by triangular fuzzy numbers and the development of the instrument panel with the two regression models (linear and power) an important contribution is made, from a theoretical and practical standpoint, of great topicality to the uncertainty management.

The two regression models come to cover a long-standing gap that requires more and more interdisciplinary contributions within economic and social processes.

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