

Exploring the Dynamics of Digital Assets through Vector Autoregressive Modeling (VAR): Implications for Fintech and Financial Systems

Andrei Cristian Spulbăr

University of Craiova, "Eugeniu Carada" Doctoral School of Economic Sciences, Romania
andrei.spulbar@gmail.com

Abstract

This study aims to explore the applicability of the VAR model in digital asset analysis, focusing in particular on Bitcoin and Ethereum, two of the most prominent and influential cryptocurrencies in the digital world. Our analysis aims not only to identify and interpret the dynamic relationships between these assets, but also to try to better understand their implications in the broader framework of financial technologies and systems. By adopting a multidisciplinary perspective, combining economic theory, statistics and digital finance, our study aspires to contribute to the existing literature and provide new insights and understanding in this fascinating and evolving field.

Key words: (Vector Autoregressive Modeling VAR), Digital Assets Analysis, Cryptocurrencies (Bitcoin and Ethereum), Fintech and Financial Systems, Time Series Analysis

J.E.L. classification: C32, G23, G41, E44, L17

1. Introduction

In an era dominated by rapid technological innovation, finance has been profoundly transformed by the emergence and evolution of digital assets such as cryptocurrencies. These new forms of assets, emblematically represented by Bitcoin and Ethereum, have not only reshaped the traditional investment perspective, but have also posed significant challenges in terms of analysing and predicting their market behaviour. In this context, financial technologies (fintech) have played a crucial role in the development and implementation of innovative analytical systems and models aimed at providing a better understanding of the dynamics of these digital assets.

One such analytical tool, the Vector Autoregressive Model (VAR), is an advanced statistical methodology that allows the examination of complex relationships between multivariate time series. Due to its flexible nature and ability to model interactions between various economic and financial variables, VAR proves to be particularly suitable for the analysis of digital finance, where the market is characterised by volatility and rapid interdependencies.

Vector Autoregression (VAR) is a statistical model used to capture the relationship between several quantities as they change over time. VAR is a type of stochastic process model. VAR models generalize the single-variable (univariate) autoregressive model by allowing multivariate time series. VAR models are often used in economics and natural sciences.

As an autoregressive model, each variable has an equation that models its evolution over time. This equation includes the lagged (prior) values of the variable, the lagged values of the other variables in the model and an error term. VAR models do not require as much knowledge about the forces influencing a variable as structural models with simultaneous equations. The only prior knowledge required is a list of variables that can be assumed to affect each other over time.

2. Literature review

Zhao, L. (2021) highlighted the crucial role of cryptocurrencies in fintech, highlighting how they reduce the dependence of financial transactions on intermediaries and contribute to the growth of the digital economy. The papers collected in the special issue of the journal Financial Innovation explore the new challenges that cryptocurrencies bring to the financial market and discuss the role of specific

data processing technologies in the fintech context. In particular, Bitcoin, as a relatively mature cryptocurrency, has sparked extensive discussion in academia, with studies focusing on its market dynamics and relationships with other financial assets.

VAR models provide a consistent and reliable approach to data description, forecasting, structural inference and policy analysis. These models differ from simultaneous equation systems in that they do not require internal-external distinction of variables in any economic theory. In addition, the presence of lagged values of dependent variables in VAR models makes it possible to make strong predictions for the future (Kumar et al. 1995). When examining the relationship between variables, it is first necessary to determine whether the variables are internal or external. Granger and Newbold (2014), examining causality between variables, suggest that when two time series cause each other, causality creates a mutual and feedback relationship Granger and Newbold (2014).

According to Watson and Teelucksingh (2002) VAR models have three basic principles:

- (a) there are no internal and external presuppositions in the system;
- (b) there are no zero constraints;
- (c) there is no rigorous underlying economic theory on which to base the model.

The main purpose of VAR modelling is not only to determine the unidirectional relationship between variables, but also to reveal the link between intermediate lag variables (Kearney and Monadjemi, 1990).

Kinal and Ratner (1982) highlight the advantages of this method of econometric analysis:

- (a) the method is simple; it is not necessary to determine which variables are internal and which are external. All variables in the VAR are endogenous;
- (b) prediction is simple; i.e. the usual LS method can be applied to each equation separately;
- (c) the estimates obtained by this method are in many cases better than those obtained from more complex models of simultaneous equations.

3. Research methodology

The first step in estimating the VAR model starts with identifying the types of variables: stationary or non-stationary. This distinction is essential in time series analysis, because for the VAR model to be built, the variables must be stationary. To check stationarity, the Dickey-Fuller test, a statistical test based on the assumption that the time series has a unit root, indicating non-stationarity, is applied. If the null hypothesis is rejected, it is concluded that the time series is stationary, which excludes significant trends or changes over time.

Next, the Dickey-Fuller test is applied to the variables ETH, BTC, RETH and RBTC to determine their stationary or non-stationary nature. The test results provide p-values for each variable, where the p-value represents the probability of obtaining a statistic under the null hypothesis. A small p-value indicates statistical significance, allowing the rejection of the null hypothesis in favour of an alternative hypothesis.

The initial calculation of a 2-lagged VAR model is essential to investigate lagged relationships between variables. It provides insight into the past influences of one variable on others, highlighting interdependencies and dynamic patterns in the observed data.

The chi-square distribution plays a crucial role in testing the autocorrelation and overall significance of the VAR model. The chi-square test for overall significance compares the test statistic with the value of the chi-square distribution at a specific degree of freedom to determine whether the model is a good fit for the data. If the test statistic exceeds the critical value from the chi-square distribution for a given significance level, the null hypothesis (that the model is not significant) can be rejected, indicating that the model has significant predictive power.

The selection of the optimal number of lags is performed using criteria such as likelihood ratio (LR), Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC), and Bayesian Schwarz information criterion (SBIC). These criteria, based on log-likelihood and number of parameters, help to find a balance between fit to the data and model complexity.

The 1 lag VAR model is used to analyse and model the relationships between endogenous variables in the system. This model requires regressing each endogenous variable on its prior and the values of the other variables.

Determining the stability of the VAR model is crucial and involves controlling for unit roots, assessing the consistency of the estimated relationships and using appropriate data. Residual analysis provides insight into model errors, and the autocorrelation test and Granger causality test are essential for assessing and quantifying dependencies between variables.

The Cholesky decomposition and Johansen test for cointegration are used to clarify the order of causality and identify long-run equilibrium relationships between variables in the system. Finally, these steps and analyses provide a sound scientific basis for interpreting the results obtained from the VAR model.

For the VAR analysis, a database downloaded from the Yahoo Finance archive was created, containing weekly closing prices for Bitcoin and Ethereum from the beginning of 2016 to the fourth week of 2023. The selection of 2016 as the starting point is motivated by two significant aspects. First, July 2016 saw the second Bitcoin halving event, an event that influences the supply and demand of the coin. Second, in 2016, Ethereum gained notoriety with the introduction of smart contracts, paving the way for various applications based on blockchain technology and increasing interest in cryptocurrencies.

Figure no. 1 Evolution of Bitcoin and Ethereum prices



Source: Author processing according to applied methodology

In the analysis, we opted for weekly data to gain a broader perspective on long-term price trends, eliminating the excessive volatility of daily data. Weekly data reduces the noise caused by temporary events by focusing on significant and sustainable price movements. They allow filtering out minor technical noise and identifying significant levels and patterns. We also assign specific variable names to represent Bitcoin prices (BTC), Ethereum prices (ETH), Bitcoin yields (RBTC), and Ethereum yields (RETH). This standardization helps track and compare price fluctuations and yields between the two cryptocurrencies, making it easier to interpret the results.

4. Findings

Table no. 1 Dickey-Fuller test for the ETH variable

Dickey-Fuller test for unit root		Number of obs = 375		
Test Statistic	Interpolated Dickey-Fuller			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	-1.600	-3.450	-2.875	-2.570

MacKinnon approximate p-value for Z(t) = 0.4837

Source: Author processing according to applied methodology

The Dickey-Fuller test statistic for the ETH variable is -1.600. Comparing it with the critical values at 1%, 5% and 10% significance levels, we observe that (-1.600) is higher than these (-3.450 at 1%, -2.875 at 5%, -2.570 at 10%). Thus, we cannot reject the null hypothesis of the existence of a unit root in the ETH variable. The p-value for Z(t) is approximately 0.4837, indicating that there is not enough statistical evidence to reject the null hypothesis. The conclusion of the Dickey-Fuller test is that the ETH variable is non-stationary and exhibits a unit root.

Table no. 2 Dickey-Fuller test for the BTC variable

Dickey-Fuller test for unit root		Number of obs = 375		
Test Statistic	Interpolated Dickey-Fuller			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	-1.449	-3.450	-2.875	-2.570

MacKinnon approximate p-value for Z(t) = 0.5586

Source: Author processing according to applied methodology

The Dickey-Fuller test statistic for the BTC variable is -1.449. Compared to the critical values at 1%, 5% and 10% significance levels, (-1.449) is higher than these (-3.450 at 1%, -2.875 at 5%, -2.570 at 10%). Therefore, we cannot reject the null hypothesis of the existence of a unit root in the BTC variable. The p-value for Z(t) is approximately 0.5586, indicating that there is not enough statistical evidence to reject the null hypothesis. The conclusion of the Dickey-Fuller test is that the BTC variable is non-stationary and exhibits a unit root.

Table no. 3 Dickey-Fuller test for the RETH variable

Dickey-Fuller test for unit root		Number of obs = 374		
Test Statistic	Interpolated Dickey-Fuller			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	-17.594	-3.450	-2.875	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

Source: author processing according to applied methodology

The Dickey-Fuller test for the RETH variable shows a statistical value (Z(t)) of -17.594. This is much lower than the critical values (-3.450 at 1%, -2.875 at 5%, -2.570 at 10%), indicating the possibility of rejecting the null hypothesis of the existence of a unit root. The p-value for Z(t) is 0.0000, providing sufficient statistical evidence for rejecting the null hypothesis. Therefore, the results indicate that the RETH variable is stationary without a unit root, suggesting no significant changes over time in its time series.

Table no. 4 Dickey-Fuller test for the RBTC variable

Dickey-Fuller test for unit root		Number of obs = 374		
		Interpolated Dickey-Fuller		
	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-18.617	-3.450	-2.875	-2.570

MacKinnon approximate p-value for Z(t) = 0.0000

Source: author processing according to applied methodology

The Dickey-Fuller test for the RBTC variable shows a statistical value (Z(t)) of -18.617, much lower than the critical values (-3.450 at 1%, -2.875 at 5%, -2.570 at 10%). This suggests rejection of the null hypothesis of the existence of a unit root in RBTC. The p-value for Z(t) is 0.0000, providing sufficient statistical evidence to reject the null hypothesis. Therefore, the results indicate that the RBTC variable is stationary without a unit root, suggesting no significant changes over time in its time series.

Table no. 5 VAR model with 2 lags
Vector autoregression

Sample:	2016w4 - 2023w12	No. of obs	=	373	
Log likelihood	= 515.0293	AIC	=	-2.707932	
FPE	= .0002286	HQIC	=	-2.666184	
Det (Sigma_ml)	= .0002166	SBIC	=	-2.602796	
Equation	Parms	RMSE	R-sq	chi2	P>chi2
reth	5	.141415	0.2174	103.6404	0.0000
rbtc	5	.105864	0.0042	1.554463	0.8170

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
reth						
reth						
L1.	.0163298	.0513668	0.32	0.751	-.0843473	.1170069
L2.	.0777722	.04595	1.69	0.091	-.0122883	.1678326
rbtc						
L1.	.6733837	.0693155	9.71	0.000	.5375277	.8092397
L2.	.0895873	.0773186	1.16	0.247	-.0619544	.241129
_cons	.0090209	.0074086	1.22	0.223	-.0054997	.0235414
rbtc						
reth						
L1.	.007687	.0384534	0.20	0.842	-.0676803	.0830543
L2.	.0060011	.0343984	0.17	0.862	-.0614185	.0734207
rbtc						
L1.	.0319393	.0518899	0.62	0.538	-.0697631	.1336416
L2.	.0462187	.0578811	0.80	0.425	-.0672261	.1596635
_cons	.0096289	.0055461	1.74	0.083	-.0012412	.0204991

Source: author processing according to applied methodology

The 2-lag VAR model was estimated based on 373 observations between week 4 of 2016 and week 12 of 2023. The results show a log-likelihood of 515.0293 and an AIC of -2.707932, indicating a moderate fit. The FPE and HQIC values are 0.0002286 and -2.666184, and the determinant of the estimated covariance matrix, Det(Sigma_ml), is 0.0002166. SBIC is -2.602796. The model includes equations for RETH and RBTC, each with 5 estimated parameters. For RETH, RMSE is 0.141415, and R-sq is 0.2174, with a significant chi-square test (p-value: 0.0000). For RBTC, RMSE is 0.105864, and R-sq is 0.0042, with an insignificant chi-square test (p-value: 0.8170). Coefficients for each lag are present, with associated standard errors, z-scores, and p-values.

while the relationship for RBTC is weaker or insignificant, evidenced by the low coefficient of determination (0.0024) and chi-square test with large p-value (0.6418).

Table no. 8 VAR model stability
 Eigenvalue stability condition

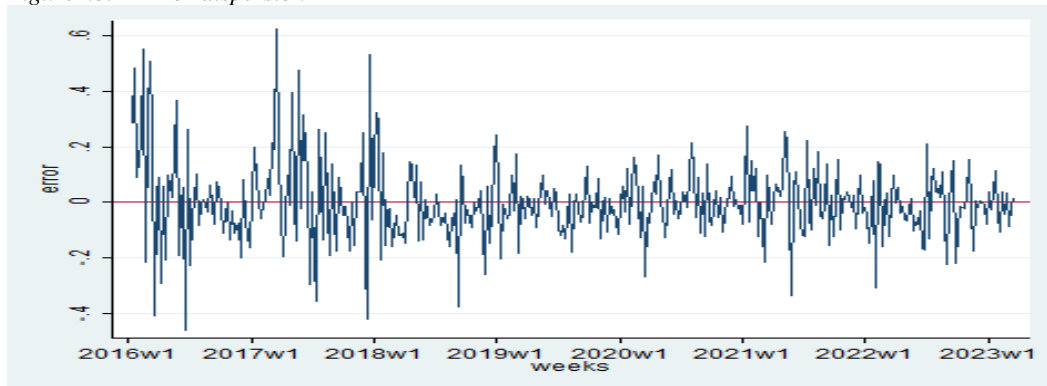
Eigenvalue	Modulus
.1631504	.16315
-.07909147	.079091

All the eigenvalues lie inside the unit circle.
 VAR satisfies stability condition.

Source: author processing according to applied methodology

The stability of the VAR model is confirmed by examining the eigenvalues: 0.16315 and 0.079091. Both are below 1, indicating that all eigenvalues are within the unit circle. Therefore, the VAR model is stable and suitable for time series analysis and forecasting of the variables involved.

Figure no. 2 Error dispersion



Source: author processing according to applied methodology

The graph shows that the model residuals are mainly concentrated around zero, reflecting significant accuracy in predictions. The moderate dispersion of the residuals shows a uniform distribution with no significant fluctuations or outliers, indicating reliability and performance in accurately predicting the results.

Table no. 9 Residue diagnosis

Variable	Obs	Mean	Std. Dev.	Min	Max
error	374	-4.73e-11	.1420806	-.4602357	.6258661

Source: author processing according to applied methodology

The VAR model residual diagnostics show a mean close to zero (-4.73e-11) and a standard deviation of 0.1420806. The residuals values are centered around zero, with moderate dispersion and no significant outliers or deviations. These results suggest a good fit of the model to the data, indicating no undesirable patterns or structures that would affect the interpretation and forecasting of the variables.

Table no. 10 Lagrange autocorrelation test

Lagrange-multiplier test

lag	chi2	df	Prob > chi2
1	3.3543	4	0.50038
2	3.2946	4	0.50979

H0: no autocorrelation at lag order

Source: author processing according to applied methodology

The autocorrelation test using the Lagrange-multiplier method showed chi-square statistics of 3.3543 and 3.2946 for the first and second lag, with degrees of freedom of 4. The associated probabilities are 0.50038 and 0.50979. Even though there is insufficient evidence to reject the null hypothesis, which supports the absence of autocorrelation at these lag orders, the test did not identify significant autocorrelation in the VAR model. This argues that the model residuals are independent and show no significant autocorrelation, validating the model in this respect.

Table no. 11 Granger causality tests

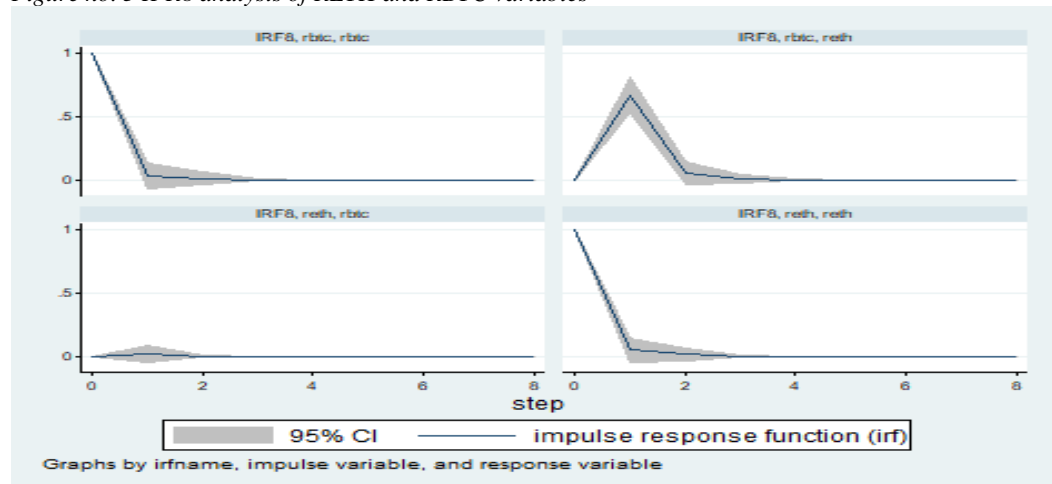
Granger causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2
reth	rbtc	92.014	1	0.000
reth	ALL	92.014	1	0.000
rbtc	reth	.40123	1	0.526
rbtc	ALL	.40123	1	0.526

Source: author processing according to applied methodology

Granger causality test results indicate a significant one-way causal relationship between RETH and RBTC variables in the VAR model. According to these results, RBTC significantly influences RETH, suggesting that Bitcoin price movements can affect Ethereum prices. In contrast, there is no significant causal relationship from RETH to RBTC, indicating that Ethereum prices do not have a significant impact on Bitcoin prices.

Figure no. 3 IFR8 analysis of RETH and RBTC variables



Source: author processing according to applied methodology

In the graph, IFR8 provides insight into the influence of exogenous variables on endogenous variables. The RBTC variable manifests a dominant and constant response with a significant influence, while the RETH variable has a minor impact on it. The results indicate that the RETH variable mainly explains its own response, with little contribution from the RBTC variable.

Table no. 12 Cholesky decomposition - response of the RETH variable
Results from IRF8

step	(1) fevd	(2) fevd
0	0	0
1	1	0
2	.804232	.195768
3	.803188	.196812
4	.803127	.196873
5	.803126	.196874
6	.803126	.196874
7	.803126	.196874
8	.803126	.196874

(1) irfname = IRF8, impulse = reth, and response = reth

(2) irfname = IRF8, impulse = rbtc, and response = reth

Source: author processing according to applied methodology

The results in the above table, based on IRF8 (Impulse Response Function), analyze how the RETH variable responds over time to perturbations introduced in the RETH and RBTC variables. Each time step shows the proportion of the response explained by each variable. For example, at step 0 there is no response, at step 1 100% of the response is attributed to the RETH variable, and the RBTC variable does not contribute. The proportions then change, with approximately 80% of the response explained by the RETH variable and 20% by the RBTC variable at step 2, continuing the same trend for subsequent steps.

Table no. 13 Cholesky decomposition - response of the RBTC variable
Results from IRF8

step	(1) fevd	(2) fevd
0	0	0
1	.006881	.993119
2	.007893	.992107
3	.007905	.992095
4	.007906	.992094
5	.007906	.992094
6	.007906	.992094
7	.007906	.992094
8	.007906	.992094

(1) irfname = IRF8, impulse = reth, and response = rbtc

(2) irfname = IRF8, impulse = rbtc, and response = rbtc

Source: author processing according to applied methodology

The results in the above table, based on IRF8 (Impulse Response Function), assess how the RBTC variable responds over time to a perturbation introduced in the RETH variable and in RBTC. Each time step shows the proportion of the response explained by each variable. At step 0, there is no response, and at step 1, approximately 99.3% of the response of the RBTC variable is explained by it, with a negligible contribution from the RETH variable. This trend persists at step 2, where almost 99.2% of the response is explained by the RBTC variable. The percentages stabilise and remain constant thereafter.

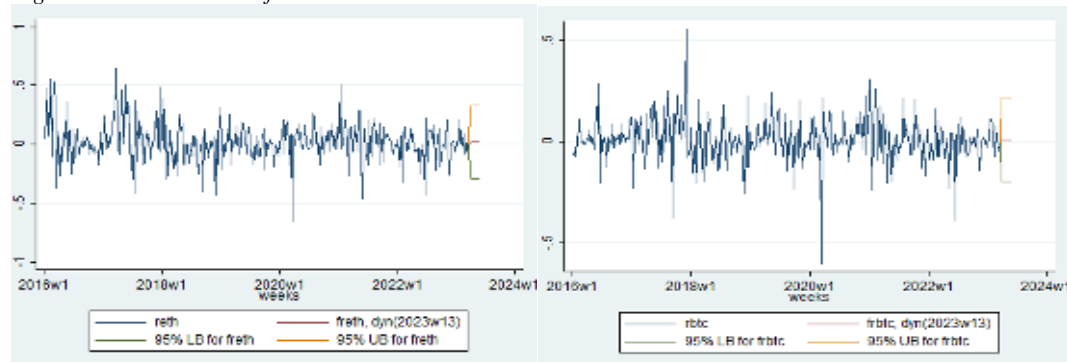
Table no. 14 Johansen tests for cointegration

Johansen tests for cointegration					
Trend: constant			Number of obs =		374
Sample: 2016w3 - 2023w12			Lags =		1
maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value
0	2	238.5893	.	548.0740	15.41
1	5	424.15514	0.62929	176.9423	3.76
2	6	512.62628	0.37694		

Source: author processing according to applied methodology

Johansen tests for cointegration, based on 374 observations and a lag of 1, show that at a 5% significance level, the maximum cointegration test indicates a rank of 0, suggesting no cointegration between variables. However, at a maximum rank of 1, the results show cointegration, indicated by a trailing statistic of 176.9423, exceeding the critical value of 3.76. Thus, there is a cointegrating relationship between variables, at least to a limited extent, at a lower significance level. The tests indicate the possibility of the existence of a stable relationship and a long-standing link between variables.

Figure no. 4 Prediction of RETH and RBTC variables



Source: author processing according to applied methodology

The future predictions for the RETH and RBTC variables, shown in the graph above, include separate confidence intervals. The interval for RETH is narrower, reflecting higher certainty, while for RBTC it is wider, indicating a higher level of uncertainty. Although based on careful analysis of the historical behaviour of these variables, these forecasts cannot provide absolute certainty in their future evolution. They provide an informed perspective while recognising the inherent unpredictability of financial markets.

5. Conclusions

Our study demonstrated the effectiveness of the Vector Autoregressive Model (VAR) in analyzing the complex relationships between Bitcoin and Ethereum. We found that although cryptocurrencies are often perceived as highly volatile and unpredictable, the use of the VAR model allows for a deeper understanding and more accurate prediction of their behavior.

The research contributes significantly to the fintech field by showing how traditional financial analysis methodologies can be applied to digital assets. Through this study, we aim to lay the foundation for a better understanding of the cryptocurrency market by integrating knowledge from diverse fields such as economics, finance and statistics.

The results highlight the need for a prudent and informed approach to cryptocurrency investment and trading. This is crucial for investors and practitioners as it provides a solid basis for decision-making in a market environment characterized by rapid change and high volatility.

Although the VAR model provided valuable insights, our study acknowledges some limitations. Cryptocurrencies are subject to complex external influences, and the VAR model cannot always fully encompass these dynamics. Thus, it is essential that future research continues to explore and improve predictive models for these digital assets.

The study paves the way for further research in fintech, with a particular focus on cryptocurrencies. There is an ongoing need to explore new models and analytical approaches to keep pace with the rapid evolution of this sector. It is also important to investigate the impact of macroeconomic and regulatory factors on cryptocurrency markets.

6. References

- Granger, C.W.J., Newbold, P., 2014. *Forecasting economic time series*. Academic Press.
- Kearney, C., Monadjemi, M., 1990. Fiscal policy and current account performance: International evidence on the twin deficits. *Journal of Macroeconomics*, 12(2), pp. 197–219. [https://doi.org/10.1016/0164-0704\(90\)90029-a](https://doi.org/10.1016/0164-0704(90)90029-a)
- Kinal, T., Ratner, J., 1986. A VAR Forecasting model of a regional Economy: Its construction and comparative accuracy. *International Regional Science Review*, 10(2), pp. 113–126. <https://doi.org/10.1177/016001768601000202>
- Kumar, V., Leone, R.P., Gaskins, J.N., 1995. Aggregate and disaggregate sector forecasting using consumer confidence measures. *International Journal of Forecasting*, 11(3), pp. 361–377. [https://doi.org/10.1016/0169-2070\(95\)00594-2](https://doi.org/10.1016/0169-2070(95)00594-2)
- Watson, P.K., Teelucksingh, S.S., 2002. *A practical introduction to econometric methods: Classical and Modern*.
- Zhao, L., 2021. The function and impact of cryptocurrency and data technology in the context of financial technology: introduction to the issue. *Financial Innovation*, 7(1). <https://doi.org/10.1186/s40854-021-00301-w>
- <https://finance.yahoo.com/quote/BTC-USD/history/>
- <https://finance.yahoo.com/quote/ETH-USD/history/>